

## INCOMPATIBILITY OF TRENDS IN MULTI-YEAR ESTIMATES FROM THE AMERICAN COMMUNITY SURVEY

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The American Community Survey (ACS) provides one-year (1y), three-year (3y) and five-year (5y) multi-year estimates (MYEs) of various demographic and economic variables for each “community,” although the 1y and 3y may not be available for communities with a small population. These survey estimates are not truly measuring the same quantities, since they each cover different time spans. Using some simplistic models, we demonstrate that comparing different period-length MYEs results in spurious conclusions about trend movements. A simple method utilizing weighted averages is presented that reduces the bias inherent in comparing trends of different MYEs. These weighted averages are nonparametric, require only a short span of data, and are designed to preserve polynomial characteristics of the time series that are relevant for trends. The basic method, which only requires polynomial algebra, is outlined and applied to ACS data. In some cases there is an improvement to comparability, although a final verdict must await additional ACS data. We draw the conclusion that MYE data is not comparable across different periods.

**1. Introduction.** The American Community Survey (ACS) replaces the former Census Long Form, providing timely estimates available throughout the decade. The ACS sample size is comparable to that of the Census Long Form; variability in the sampling error component of the ACS is partially reduced through a rolling sample [Kish (1981)]. The rolling sample refers to the pooling of sample respondents over time—in some cases this may be viewed as an approximate temporal moving average of single period estimates. In particular, estimates from regions with at least 65,000 people are produced with a single year of data, whereas if the population is between 20,000 and 65,000, then three years of data are combined, and if the population is less than 20,000, then five years of data are pooled. A somewhat dated overview of the ACS can be found in Alexander (1998). More current details can be found in the Census Bureau (2006) and Torrieri (2007).

In order to examine longer time series of ACS data, it is necessary to examine older estimates published for a small group of regions in the Multi-Year Estimates Study (MYES), which is publicly available at [www.census.gov/acs/www/AdvMeth/Multi\\_Year\\_Estimates/online\\_data\\_year.html](http://www.census.gov/acs/www/AdvMeth/Multi_Year_Estimates/online_data_year.html).

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The MYES was a trial study for the ACS that produced one, three and five year estimates for counties included in the 1999–2001 demonstration period and their constituent geographies, using data from 1999 through 2005. The Multi-Year Estimates (MYEs) are divided according to period-length—either one-year (1y),<sup>1</sup> three-year (3y) or five-year (5y)—the time period, the county and the geographic type within the county (e.g., school district). There are hundreds of variables available, which are broken into four categories: demographic, economic, social and housing. Most of the variables are totals, averages, medians or percentiles.

Because some counties have a low population, it was deemed desirable by the U.S. Census Bureau to decrease sampling error for smaller geographies and subpopulations by using a rolling sample; a discussion of issues associated with this methodology can be found in the National Academy of Sciences Panel on the Functionality and Usability of Data from the American Community Survey [Citro and Kalton (2007)]. In essence, responses over a 3y or even a 5y span are gathered together into one database, and a statistic of interest is computed over the temporally enlarged sample. In many cases, this is approximately equal to computing a simple moving average of 1y estimates. This is known as a rolling sample—see Kish (1981, 1998) and Alexander (2001) for a discussion. For larger counties, the 1y MYE would be available as well. The question of whether each year should be equally weighted was addressed in Bell (1998) and Breidt (2007); since all the responses are pooled in the 3y and 5y cases, the U.S. Census Bureau judged that it would be impractical to use some alternative weighting scheme (such as weighting the most recent year of data more highly). Hence, the MYEs are formed from contributions over multiple years that are *equally* weighted. Although this approach is simple, one repercussion is that some lag (or time delay) is induced by the use of rolling samples (whereas an unequal weighting scheme can be devised such that time delay is reduced or eliminated for certain components of the time series).

The time delay effect is easy to understand in the case that the data is a simple polynomial, such as a line or a quadratic. In the former case, a three-period average induces a time delay of exactly one time unit, whereas the five-period average delays the line by two time units. For higher degree polynomials the delay is not exact, and yet visually there is a definite shift in the graph of one or two units. Assuming that trends in ACS MYEs are locally given by low-degree polynomials, this brief discussion illustrates the problem with comparing MYEs of different period lengths (and this is further expounded in Sections 2 and 3 below). In particular, making comparisons across regions of MYEs of different period lengths will in general lead to false conclusions and spurious deductions, and therefore should be avoided. This paper assesses the extent of this problem through some extremely simple models, and proposes a class of trend-preserving weighted averages that can be used to illustrate and identify the sorts of false conclusions arising

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<sup>1</sup>Technically, the 1y are not MYEs, but we will ignore this for didactic purposes.

from such inter-period comparisons. The perspective of this author is that such cross-period MYE comparisons should not be made for reasons discussed in the subsequent sections. Although use of the proposed weighted averages in this paper may well, in some cases, reduce the quantity of spurious conclusions drawn from the data, it is acknowledged that they do not provide a full solution to the problem of incomparability.

In Section 2 we provide additional discussion of the construction of MYEs, explicating the practical factors militating against inter-period comparisons. Then in Section 3 we discuss a simple model for MYEs that focuses on the temporal aspects, while ignoring sampling error for simplicity. Using this formal approach, we can illustrate in a quantitative fashion the pitfalls that may occur from making cross-period MYE comparisons. In Section 4 we propose a system of weighted averages that preserve any local polynomial trends, ensuring that these trends for 1y, 3y and 5y are identical after application of the weights. This is a general technique based on simple time series analysis and polynomial algebra, and we apply it in the linear trend case to MYE data in Section 5, making use of the newly available ACS data extended by the trial period of the MYES. Through several examples, we illustrate the dangers of making inappropriate comparisons, that is, cross-region comparisons involving MYEs of different period lengths. Finally, Section 6 summarizes the results of the paper and the main difficulties in inter-period comparisons.

**2. Practical issues in making comparisons.** Beyond the issues of time delay raised in the Introduction and further described below, there is a problem comparing MYEs of different period lengths due to the differences in how the estimates are constructed. A detailed discussion of these issues is beyond the scope of this paper [for more information the reader is referred to Fay (2007), Starsinic and Tersine (2007), and Tersine and Asiala (2007)], but here we briefly highlight some relevant points.

In the construction of MYEs a weighting method is used that is different for 1y versus 3y and 5y. In the former case, baseweights are used that are defined as the inverse of sampling probabilities, with some differences between Housing Units (HU) and Group Quarters (GQ). Next, there is a nonresponse adjustment followed by the application of controls to a set of independent HU estimates derived from the U.S. Census Bureau's Population Estimates Program (GQs are handled with separate controls). For the 3y and 5y estimates, similar weighting and adjustments are made, but based off of data pooled over the whole three years and five years respectively. Moreover, housing unit controls are further modified by the so-called *g*-weighting (a type of calibration) [see Fay (2005, 2006, 2007)], with the objective of reducing (sampling error) variances at the sub-county aggregation level. This process involves linking administrative records data with the ACS sampling frame [Starsinic and Tersine (2007)].

As a result of *g*-weighting, the 3y and 5y estimates are fundamentally different in their construction from the 1y. We also point out that, apart from the *g*-weighting, there is also the issue of additional pooling in 3y and 5y prior to weighting and nonresponse adjustment; thus, a 5y estimate will have effectively five times as many sample cases receiving weighting over the 1y estimate. Furthermore, the population controls will vary between MYEs, since the vintage of the population estimates will correspond to the final year in the particular MYE. So the 3y MYE for 2005, 2006 and 2007 is controlled to the average population for those years at a 2007 population vintage, whereas the 1y MYE for each of the corresponding years 2005, 2006 and 2007 will each be based off population vintages from those three years; this further interferes with comparability. A related issue is inflation adjustment for monetary variables, which is handled by controlling to dollars in the latest year of the period.

These are fundamental incompatibilities; one may see that 1y, 3y and 5y are really measuring different quantities. The weighted average methodology of this paper—presented below—can address the issue of pooling in an approximate fashion, but does not provide a resolution to the effects of *g*-weighting, nonresponse adjustment and variable (population and monetary) vintages. However, given that it is common in trend analysis of demographic and economic time series to compare data that have no common basis of measurement [e.g., consumption versus income is analyzed for co-integration in Engle and Granger (1987)], it is only vital to account for time delay shifts in the respective time series. Although such weighted MYEs are not strictly comparable, they can still be used as subjects in such a longitudinal or multivariate analysis, just as similar situations are treated throughout the social sciences [see Granger (2004)].

**3. Comparing MYEs.** This section develops the issue of comparability in a mathematical framework, so that we can obtain a quantitative view of why inter-period comparisons are problematic. The MYEs are currently available as an annual time series, and we use the notation  $Y_t^{(k)}$  for the *ky* MYE available at year *t*, where  $k = 1, 3, 5$ . We define the Simple Moving Average (SMA) polynomial of order *k* by

$$\Theta^{(k)}(z) = \frac{1}{k}(1 + z + \cdots + z^{k-1}).$$

As usual, *B* denotes the backshift operator. Because of the method of construction of the MYEs described in Section 1, we might think that  $Y_t^{(5)} = \Theta^{(5)}(B)Y_t^{(1)}$  and  $Y_t^{(3)} = \Theta^{(3)}(B)Y_t^{(1)}$  are approximately true equations [such an assumption is used for certain variance calculations in Citro and Kalton (2007)]. However, in our experience this approximation is poor for many variables, and is fair for only a few variables—typically those involving linear statistics such as totals and averages.

Therefore, we adopt the following error model for the purpose of demonstrating issues of comparability of trends:

$$(1) \quad Y_t^{(k)} = \Theta^{(k)}(B)\mu_t + \varepsilon_t^{(k)},$$

for  $k = 1, 3, 5$ . Here  $\mu_t$  is a common deterministic trend function, and the errors  $\varepsilon_t^{(k)}$  include sampling error, serially correlated stochastic trend perturbations and “nonadditive error,” that is, the error attributed to assuming a moving average relationship to be valid. We will not be concerned with the statistical properties of these errors, though they are assumed to be identically distributed in  $t$  with mean zero. The common trend  $\mu_t$  is conceived of abstractly, and does not necessarily have a fundamental interpretation in terms of the population trend. Although other models could be considered [such as  $Y_t^{(k)} = \Theta^{(k)}(B)(\mu_t + \varepsilon_t^{(k)})$ ], (1) will be sufficient for our illustrative purposes.

Now suppose that we have two time series of MYEs, denoted  $Y_t^{(k)}$  (with trend  $\mu_t^Y$  and error process  $\varepsilon_t^{(k)}$ ) and  $Z_t^{(k)}$  (with trend  $\mu_t^Z$  and error process  $\eta_t^{(k)}$ ). These MYEs may correspond to two different geographical regions, and a practitioner may be interested in comparing the trends  $\mu_t^Y$  and  $\mu_t^Z$ , either at several time points or perhaps at just one time  $t_0$ . Formally, we might consider the following hypotheses, although many others are conceivable:

$$H_0: \mu_{t_0}^Y = \mu_{t_0}^Z,$$

$$H_a: \mu_{t_0}^Y > \mu_{t_0}^Z.$$

In this formulation, the values of the mean at time  $t_0$  simply become parameters, and it is the statistician’s task to devise parameter estimates that are accurate and precise. Since typically in applications it is desirable to make trend comparisons in real-time, any estimators must be a function of present and past data only, that is,  $\hat{\mu}_{t_0}^Y$  and  $\hat{\mu}_{t_0}^Z$  are functions of the MYE series at times  $t_0, t_0 - 1, \dots$ . The simplest unbiased estimators are  $\hat{\mu}_{t_0}^Y = Y_{t_0}^{(1)}$  and  $\hat{\mu}_{t_0}^Z = Z_{t_0}^{(1)}$ , but the 1y MYEs are not always available. Suppose that the first region ( $Y$ ) includes 1y, 3y and 5y period MYEs, but the second ( $Z$ ) includes only 3y and 5y.

Commonly, users of MYEs (despite official cautions to the contrary) will take  $\hat{\mu}_{t_0}^Y = Y_{t_0}^{(1)}$  and  $\hat{\mu}_{t_0}^Z = Z_{t_0}^{(3)}$  [or even equal to  $Z_{t_0}^{(5)}$ ], even though the latter is a biased estimate [due to the phase delay of  $\Theta^{(3)}(B)$ ; see below] of the trend. We refer to this as the “inapt” comparison. Seeking to mitigate the phase delay, we can put both trend estimates on an equal footing by taking  $\hat{\mu}_{t_0}^Y = Y_{t_0}^{(3)}$  and  $\hat{\mu}_{t_0}^Z = Z_{t_0}^{(3)}$ . Now both trend estimates are biased, but at least they are biased in a similar fashion; this will be called the “untimely” comparison. A “proper” comparison is one in which both estimates are unbiased for their respective trend values. Of course, even for a proper comparison Type I and II errors will occur due to statistical uncertainty, but at least the bias will be eliminated.

One could test the hypothesis of equal trends via  $\hat{\mu}_{t_0}^Y - \hat{\mu}_{t_0}^Z$ ; this has the following expectation for the inapt comparison:  $\mu_{t_0}^Y - (\mu_{t_0}^Z + \mu_{t_0-1}^Z + \mu_{t_0-2}^Z)/3$ , which need not be zero under  $H_0$ . For the untimely comparison, the expectation would be

$$((\mu_{t_0}^Y - \mu_{t_0}^Z) + (\mu_{t_0-1}^Y - \mu_{t_0-1}^Z) + (\mu_{t_0-2}^Y - \mu_{t_0-2}^Z))/3.$$

If the trends agree at times  $t_0$ ,  $t_0 - 1$ , and  $t_0 - 2$ , this quantity is zero; however, some bias is to be expected under  $H_0$ . In contrast, it is clear from the definition of the proper comparison that the mean of  $\hat{\mu}_{t_0}^Y - \hat{\mu}_{t_0}^Z$  is zero under  $H_0$ .

From this discussion, we see that making inferences about trends based on a direct use (i.e., by looking just at the values rather than some more complicated statistics) of MYEs of different period lengths leads to bias even in the case that a highly idealized model holds true. The incidence of spurious conclusions (i.e., Type I errors) can be reduced by making proper comparisons, and we explore this further in the following section. However, even proper comparisons have their limitations, and our attitude is that MYEs of different period length should not be compared; using a proper comparison provides an improvement, but false conclusions can still be obtained (not to speak of the practical issues raised in Section 2).

We note that the incomparability of trends increases with the dispersion of the errors  $\varepsilon_t^{(k)}$ ; if these errors were zero, then the rolling sample would be exactly a moving average, and a proper comparison would enable full comparability of MYE trends. A crude assessment of the size of these errors, relative to the trend, is given by the ‘‘Noise-Signal Ratio’’ (NSR)

$$\frac{\varepsilon_t^{(k)}}{\Theta^{(k)}(B)\mu_t} = \frac{Y_t^{(k)}}{\Theta^{(k)}(B)\mu_t} - 1.$$

This is only well-defined when  $\Theta^{(k)}(B)\mu_t$  is nonzero, and we generally suppose that it is positive at all times. Since we do not know  $\mu_t$ , we can substitute  $Y_t^{(1)}$  when the 1y MYEs are available. Then for  $k = 3, 5$ , we have  $Y_t^{(k)}/\Theta^{(k)}(B)Y_t^{(1)} - 1$  as our estimate of the NSR. For convenience, we will instead use logarithms of noise and signal, which are approximated (by first-order Taylor series) by the former expression:

$$NSR_t^{(k)} = \log Y_t^{(k)} - \log \Theta^{(k)}(B)Y_t^{(1)}$$

for  $k = 3, 5$ . Computing this quantity at all available times  $t$ , we define a compatibility measure by

$$C^{(k)} = \max_t |NSR_t^{(k)}|.$$

If this measure is small, for example,  $C^{(k)} = 0.01$ , then the rolling sample is well-approximated by a moving average, and the proper comparison is more meaningful.

**4. Trend-preserving weighted averages.** In what follows, the function of the model (1) is to illustrate the incomparability of MYEs of different period length; we are not interested in fitting the model to actual MYEs in order to pursue statistical inference. In this sense, the model only serves a pedagogical purpose. Next, suppose that  $\mu_t$  is given by a polynomial of degree  $d$  in  $t$ . Is it possible to find sets of weighted averages, or linear filters, such that when applied to each MYE the trends will coincide? That is, if we view the underlying trend of the  $ky$  MYE as  $\Theta^{(k)}(B)\mu_t$ , then we seek three filters  $\Psi^{(k)}(B)$  such that  $\Psi^{(k)}(B)\Theta^{(k)}(B)\mu_t$  is the same for each  $k = 1, 3, 5$ ; or, in other words,

$$(2) \quad \Psi^{(1)}(z) = \Psi^{(3)}(z)\Theta^{(3)}(z) = \Psi^{(5)}(z)\Theta^{(5)}(z).$$

Since users are typically interested in comparisons utilizing the most current data available, it makes sense to formulate our problem with concurrent filters, that is, filters that only depend on present and past data. Therefore, each filter is of the form

$$\Psi^{(k)}(z) = \sum_{j \geq 0} \psi_j^{(k)} z^j.$$

In practice, only a finite number of the coefficients  $\psi_j^{(k)}$  are nonzero. Now a filter  $\Psi(z)$  will pass (i.e., leave invariant) a polynomial of degree  $d$  if  $\Psi(1) = 1$  and  $\frac{\partial^j}{\partial z^j} \Psi(z)|_{z=1} = 0$  for  $1 \leq j \leq d$  [Brockwell and Davis (1991), page 39]. Now using (2) and the fact that  $\Theta^{(3)}(z)$  and  $\Theta^{(5)}(z)$  share no common roots, it is easy to see that

$$\Psi^{(1)}(z) = \Phi(z)\Theta^{(3)}(z)\Theta^{(5)}(z).$$

We are free to design the polynomial  $\Phi(z)$  such that the polynomial-passing constraints are satisfied; hence,  $\Phi(z)$  must have degree at least  $d$ . The following theorem describes how to construct this polynomial.

**THEOREM 1.** *The minimal length concurrent filters  $\Psi^{(k)}$  that pass degree  $d$  polynomials and satisfy (2) are given by*

$$\begin{aligned} \Psi^{(5)}(z) &= \Phi(z)\Theta^{(3)}(z), \\ \Psi^{(3)}(z) &= \Phi(z)\Theta^{(5)}(z), \\ \Psi^{(1)}(z) &= \Phi(z)\Theta^{(3)}(z)\Theta^{(5)}(z), \end{aligned}$$

where the coefficients of  $\Phi(z)$  are given by the first column of the inverse of the matrix with entry  $jk$  given by

$$\left. \frac{\partial^{j-1}}{\partial z^{j-1}} [z^{k-1}\Theta^{(3)}(z)\Theta^{(5)}(z)] \right|_{z=1}.$$

PROOF. Let  $\Theta(z) = \Theta^{(3)}(z)\Theta^{(5)}(z)$ , with  $\phi_k$  the coefficients of  $\Phi(z)$ . Applying the polynomial-passing constraints yields

$$\begin{aligned} 1_{\{j=0\}} &= \sum_{l=0}^j \binom{j}{l} \frac{\partial \Phi(z)}{\partial z^l} \Big|_{z=1} \frac{\partial \Theta(z)}{\partial z^{j-l}} \Big|_{z=1} \\ &= \sum_{l=0}^j \binom{j}{l} \sum_{k=0}^d \phi_k \frac{k!}{(k-l)!} \frac{\partial \Theta(z)}{\partial z^{j-l}} \Big|_{z=1} \\ &= \sum_{k=0}^d \phi_k \frac{\partial^j}{\partial z^j} [z^k \Theta(z)] \Big|_{z=1}. \end{aligned}$$

This is easily rewritten in matrix form, from which the result follows.  $\square$

EXAMPLE (Linear trends). Supposing that the trend is linear and  $d = 1$ , we have

$$\begin{aligned} \Psi^{(5)}(z) &= (4 + z + z^2 - 3z^3)/3, \\ \Psi^{(3)}(z) &= (4 + z + z^2 + z^3 + z^4 - 3z^5)/5, \\ \Psi^{(1)}(z) &= (4 + 5z + 6z^2 + 3z^3 + 3z^4 - z^5 - 2z^6 - 3z^7)/15. \end{aligned}$$

EXAMPLE (Quadratic trends). Supposing that the trend is quadratic and  $d = 2$ , we have

$$\begin{aligned} \Psi^{(5)}(z) &= (26 - 11z + 3z^2 - 23z^3 + 14z^4)/9, \\ \Psi^{(3)}(z) &= (26 - 11z + 3z^2 + 3z^3 + 3z^4 - 23z^5 + 14z^6)/15, \\ \Psi^{(1)}(z) &= (26 + 15z + 18z^2 - 5z^3 + 9z^4 - 17z^5 - 6z^6 - 9z^7 + 14z^8)/45. \end{aligned}$$

Theorem 1 has the following interpretation. If one wishes to make a proper comparison of MYEs (defined in Section 3) that preserves polynomials of order  $d$ , then the minimal length linear filters that accomplish this goal are given by Theorem 1.

**5. Illustrations on ACS data.** We now provide three illustrations of the concepts discussed in this article. We focus on *Median Household Income* in Pima, AZ, *Number of Divorced Males* in Lake, IL, and *Median Age* in Hampden, MA. These three counties are included in the MYES and, therefore, the data extends back to the year 2000. In particular, the following MYEs are available: 2000 through 2007 for 1y, 2001 through 2005 and 2007 for 3y, and 2003 through 2005 for 5y. The year index here refers to the last year that entered into the sample, and so is consistent with our notation for  $Y_t^{(k)}$ . Current ACS estimates are now available for all geographical regions, covering the 1y years 2006 and 2007, and the 3y MYE 2005–2007 has just become available. Letting  $t$  range between 00 and 05 (referring to the year), the available database is

$Y_{00}^{(1)}, \dots, Y_{07}^{(1)}, Y_{01}^{(3)}, \dots, Y_{05}^{(3)}, Y_{07}^{(3)}, Y_{03}^{(5)}, \dots, Y_{05}^{(5)}$ . In order to apply our methods, we need to impute (by forecasting) the 3y MYE  $Y_{06}^{(3)}$  and the 5y MYEs  $Y_{06}^{(5)}$  and  $Y_{07}^{(5)}$ . (This is a provisional necessity, since in the future full time series data for all counties will be published.)

The missing values are obtained by forecasting them utilizing a simple random walk model, which is feasible for these time series based on economic and demographic considerations (to actually fit a time series model to such a short series is pointless):

$$\begin{aligned}\widehat{Y}_{06}^{(3)} &= \frac{1}{2}(Y_{05}^{(3)} + Y_{07}^{(3)}), \\ \widehat{Y}_{06}^{(5)} &= Y_{05}^{(5)} + \frac{1}{2}(Y_{05}^{(5)} - Y_{03}^{(5)}), \\ \widehat{Y}_{07}^{(5)} &= Y_{05}^{(5)} + \frac{2}{2}(Y_{05}^{(5)} - Y_{03}^{(5)}).\end{aligned}$$

The MYEs (with imputed values in bold) are given in Table 1. The final row of the table gives the various 2007 trend values estimated via the method of Section 4 [the data and calculations are given in McElroy (2009)]. Note that  $Y_{01}^{(3)}$  and  $Y_{03}^{(5)}$  are not used in the calculation of these trend estimates. Although the Income MYEs follow a linear growth pattern, the Divorce MYEs fluctuate more in their slope component, whereas the Age MYEs trend upward very slowly with little noise. Thus, we might say that Income and Age exhibit linear trend lines, whereas Divorce is nonlinear; it is important to consider different types of trend behavior in order to evaluate this paper's method.

As far as the linear approximation to the rolling sample, we can compute the NSR comparability measure for years 2002–2007 for  $k = 3$ , and 2004–2007 for  $k = 5$  (by including the forecasted data). For Income  $C^{(3)} = 0.017$  and  $C^{(5)} =$

TABLE 1  
*MYEs for Income, Divorce and Age. Estimates have been forecast extended for the years 06 and 07, written in bold*

Year	Income MYEs			Divorce MYEs			Age MYEs		
	1y	3y	5y	1y	3y	5y	1y	3y	5y
00	35223			14043			36.40		
01	35615	35956		14376	14429		37.30	36.80	
02	37638	36780		17866	15504		37.00	36.80	
03	37818	37373	37510	17398	16772	15473	37.10	37.00	36.70
04	38800	38739	38608	15632	17156	15903	37.20	37.10	36.90
05	41521	40404	40055	14591	15889	15945	37.40	37.30	37.20
06	42984	<b>42395</b>	<b>41328</b>	20941	<b>17371</b>	<b>16181</b>	37.40	<b>37.35</b>	<b>37.45</b>
07	43546	44386	<b>42600</b>	21844	18852	<b>16417</b>	37.60	37.40	<b>37.70</b>
Trend	43570	45223	45320	19331	19217	16695	37.59	37.59	38.25

0.020, indicating some incompatibility. For the Divorce variable  $C^{(3)} = 0.008$  and  $C^{(5)} = 0.042$ , indicating a high amount of incomparability (though most of this comes from the portion of the data that is forecasted, and thus might be resolved when the real numbers are published). Finally, the Age variable is highly compatible with  $C^{(3)} = 0.002$  and  $C^{(5)} = 0.004$ .

Now imagine having two replications of each variable for two separate regions: county A with all period-length MYEs available, and county B with a lower population such that only 3y and 5y MYEs are available. Starting with the Divorce variable, an illustration of the time delay properties of MYEs is provided in comparing 1y to one-year-ahead-3y MYEs; there is a fairly close match up until the 2005 1y MYE and 2006 3y MYE. However, this latter value is imputed, and the true value could easily have decreased from 2005; instead the imputation increases merely because there is so much gain in the 2007 3y MYE. The 2007 “inapt” comparison discussed in Section 3 would then compare 21,844 with 18,852 or 16,417; these are  $-13.7\%$  and  $-24.8\%$  discrepancies. If we use weighted averages for comparing trends, the discrepancies are reduced to  $-0.59\%$  and  $-13.6\%$  respectively (though given the nonlinear nature of the trend, we expect the forecasts to be inappropriate, and hence not as much emphasis should be placed on the 5y MYEs). In this case the weighted average methodology helps to properly align the series.

For the Income and Age time series data, which both exhibit linear trends (with the former having much more variability), the weighted average method can actually increase discrepancies. In the former case, the discrepancies of  $1.9\%$  and  $-2.2\%$  become  $3.8\%$  and  $4.0\%$ ; but for Age the discrepancies of  $-0.53\%$  and  $0.27\%$  become  $0\%$  and  $1.8\%$  after using weighted averages. The Age data is very stable, and here an inapt comparison indicates no change. We have not analyzed these percentages statistically, as this would require actual modeling of the time series. Nevertheless, a rough idea about trend comparability can be deduced by the discussion here.

In summary, we see through these examples that the weighted average methodology can either increase or decrease discrepancies in some cases, and seems to work less well with 5y versus 3y MYEs (although this may also be an artifact of two imputations in the 5y MYEs). Part of this increase in discrepancy is due to the weighted averages increasing the overall variance (even if they reduce the bias of direct comparisons, as discussed in Section 3); if in (1) we make the crude assumption that the errors  $\varepsilon_t^{(k)}$  are i.i.d., then the linear weights inflate the variance by a factor of 1.16 and 3 respectively for the 3y and 5y MYEs. For the 1y MYE the variance is multiplied by 0.48, but of course this MYE has the greatest variability since its sampling error component is largest. This variance inflation can be corrected by imposing extra conditions on the filter coefficients, but the result would be an even longer set of weights. It can also be observed that the random walk model used for forecasting is poorly suited to the Divorce data, since the change in direction from 2003 to 2004 in the 1y MYE is not reflected in the corresponding

time-delayed 5y MYEs of 2005–2006. A more definitive study would not rely on imputations, and would be concerned with the qualitative aspects of trends produced by weighted averages; such a study must wait at least five years due to the current ACS publication schedule.

**6. Conclusion.** The aim of this paper is first to discuss the challenges in comparing cross-period MYEs. Due to the way in which MYEs are constructed, it is apparent that 1y, 3y and 5y MYEs are different time series—and not just time-lagged or smoothed versions of some underlying series; they are estimates of different fundamental quantities (see Section 2). Nevertheless, this fact does not preclude a user from making cross-period comparisons, any more than it would be forbidden to search for common trends in economic or demographic data. Therefore, the second aim of this paper is to quantitatively assess what sorts of mathematical and statistical problems will arise in such comparisons (see Sections 3 and 4). As a third aim, the weighted averages method can be used to reduce the bias inherent in such cross-period comparisons [under certain quasi-linear assumptions such as (1)]; even so, the statistical variation in MYEs is such that sizeable discrepancies can still crop up, as demonstrated in Section 5.

In summary, the author wishes to echo the strong cautions against making cross-period comparisons issued by the U.S. Census Bureau [see Beaghen and Weidman (2008) and Citro and Kalton (2007)]. At this point the weighted average methodology mainly serves to identify fairly egregious types of false conclusions derived from such unwarranted comparisons, but perhaps it can also serve as a building block for future work on comparability and usability issues in the ACS.

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#### SUPPLEMENTARY MATERIAL

**Income, Divorce and Age Data with Trend Calculations** (DOI: [10.1214/09-AOAS259SUPP](https://doi.org/10.1214/09-AOAS259SUPP); .zip). This file contains the Income, Divorce and Age data of Table 1 in Excel format. Also provided are the linear trend weighted averages along with compatibility measures NSR, encoded as Excel formulas.

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