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GENERALIZED DERIVATIONS WITH ANNIHILATOR CONDITIONS IN PRIME RINGS

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Abstract. Let R be a noncommutative prime ring with its Utumi ring of quotients U, $C = Z(U)$ the extended centroid of R, F a generalized derivation of R and I a nonzero ideal of R. Suppose that there exists $0 \neq a \in R$ such that $a(F([x, y])^n - [x, y]) = 0$ for all $x, y \in I$, where $n \geq 2$ is a fixed integer. Then one of the following holds:

1. char $(R) \neq 2$, $R \subseteq M_2(C)$, $F(x) = bx$ for all $x \in R$ with $a(b - 1) = 0$ (In this case n is an odd integer);
2. char $(R) = 2$, $R \subseteq M_2(C)$ and $F(x) = bx + [c, x]$ for all $x \in R$ with $a(b^n - 1) = 0$.

1. INTRODUCTION

Let R be an associative prime ring with center $Z(R)$. Let U be the Utumi quotient ring of R. Then $C = Z(U)$ is called the extended centroid of R. Recall that a ring R is prime, if for any $a, b \in R$, $aRb = 0$ implies either $a = 0$ or $b = 0$. For $x, y \in R$, the commutator of x, y is denoted by $[x, y]$ and defined by $[x, y] = xy - yx$. By a derivation of R, we mean an additive mapping $d : R \to R$ such that $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. An additive mapping $F : R \to R$ is called a generalized derivation, if there exists a derivation d of R such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. Basic examples for generalized derivation are the mappings of the type $x \to ax + xb$ for some $a, b \in R$, which are called inner generalized derivations.

In [4], Daif and Bell proved that in a semiprime ring R if $d([x, y]) \pm [x, y] = 0$ holds for all $x, y \in K$, where d is a derivation of R and K is a nonzero ideal of R, then $K \subseteq Z(R)$.

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After that in [16], Quadri et al. studied the situation replacing derivations d by generalized derivations F. They proved that a prime ring R will be commutative if $F([x, y]) \pm [x, y] = 0$ holds for $x, y \in I$, where I is a nonzero ideal of R and F is generalized derivation of R.

More recently in [5], De Filippis and Huang investigated the situation $F([x, y])^n = [x, y]$ for all $x, y \in I$, where $n \geq 1$ is a fixed integer. They proved the following:

> *Let R be a prime ring, I a nonzero ideal of R and n a fixed positive integer. If R admits a generalized derivation F associated with a derivation d such that $F([x, y])^n = [x, y]$ for all $x, y \in I$, then either R is commutative or $n = 1, d = 0$ and F is the identity map on R.*

In the present paper, we consider the situation taking annihilating condition that is $a(F([x, y])^n - [x, y]) = 0$ for all $x, y \in I$, where $n \geq 1$ is a fixed integer.

For $n = 1$, above situation becomes $aG([x, y]) = 0$ for all $x, y \in R$, where $G(x) = F(x) - x$ for all $x \in R$ is a generalized derivation of R. Then by [6], we conclude that $G(x) = qx$ for some $q \in U$ with $aq = 0$, that is $F(x) = (q + 1)x$ for all $x \in R$, with $aq = 0$.

Therefore, we study the above situation when $n \geq 2$.

2. Main Results

First we fix a remark.

Remark. Let R be a prime ring and U be the Utumi quotient ring of R and $C = Z(U)$, the center of U (see [2] for more details). It is well known that any derivation of R can be uniquely extended to a derivation of U. In [13, Theorem 3], T.K. Lee proved that every generalized derivation g on a dense right ideal of R can be uniquely extended to a generalized derivation of U. Furthermore, the extended generalized derivation g has the form $g(x) = ax + d(x)$ for all $x \in U$, where $a \in U$ and d is a derivation of U.

Lemma 2.1. *Let $R = M_2(K)$ be the set of all 2×2 matrices over a field K and $a, b, p \in R$. If $p \neq 0$ such that $p((a[x, y] + [x, y]b)^n - [x, y]) = 0$ for all $x, y \in R$, where $n \geq 2$ a fixed integer, then one of the following holds:*

(1) *char $(R) \neq 2$, $b \in Z(R)$ and $p(a + b - 1) = 0$ (In this case n is odd integer);*

(2) *char $(R) = 2$ and $p((a + b)^n - 1) = 0$.*

Proof. By hypothesis, we have

$$p((a[x, y] + [x, y]b)^n - [x, y]) = 0 \tag{1}$$

for all $x, y \in R$.

Case-I: *Let char* $(R) = 2$.

In this case assuming $x = e_{12}$, $y = e_{21}$, we have $0 = p((a[x, y] + [x, y]b)^n - [x, y]) = p((aI_2 + I_2b)^n - I_2) = p((a + b)^n - 1)$.

Case-II: *Let char* $(R) \neq 2$.

If n is even integer, replacing y with $-y$ in (1) and then subtracting from (1), we have $2p[x, y] = 0$, that is $p[x, y] = 0$ for all $x, y \in R$. Now assuming $x = e_{12}$ and $y = e_{22}$, we have $0 = pe_{12}$ which implies $p_{11} = p_{21} = 0$. Similarly, assuming $x = e_{21}$ and $y = e_{11}$, we can prove that $p_{22} = p_{12} = 0$, that is $p = 0$, contradiction. Hence n must be odd integer.

We may assume p is not invertible, since if p is invertible, by (1) we get

$$(a[x, y] + [x, y]b)^n = [x, y]$$

for all $x, y \in R$. Then a contradiction follows by [5, Theorem 1]. Note that

$$Rp((a[x, y] + [x, y]b)^n - [x, y]) = 0$$

for all $x, y \in R$. Since R is von Neumann regular, there exists an idempotent element $e \in R$ such that $Rp = Re$. Hence we may assume that p is an idempotent element of R. As p is not invertible, Rp is a proper left ideal of R. Since any two proper left ideals are conjugate, there exists an invertible element $t \in R$ such that $Re_{11} = tRpt^{-1} = Rtpt^{-1}$, and so replacing p by tpt^{-1}, a by tat^{-1} and b by tbt^{-1}, our identity becomes

$$e_{11}((a'[x, y] + [x, y]b')^n - [x, y]) = 0 \tag{2}$$

for all $x, y \in R$, where $a' = tat^{-1}$ and $b' = tbt^{-1}$. Write $b' = \sum_{i,j=1}^{2} b'_{ij}e_{ij}$. Let $[x, y] = e_{12}$ in (2) and multiply right by e_{12}. Then we get $0 = e_{11}((a'e_{12} + e_{12}b')^n - e_{12})e_{12} = e_{11}(e_{12}b')^n e_{12} = b'^n_{21}e_{12}$. Thus $b'_{21} = 0$. Let φ and χ be two inner automorphism defined by $\varphi(x) = (1 + e_{21})x(1 - e_{21})$ and $\chi(x) = (1 - e_{21})x(1 + e_{21})$. Then we have

$$\varphi(e_{11})((\varphi(a')[x, y] + [x, y]\varphi(b'))^n - [x, y]) = 0 \tag{3}$$

for all $x, y \in R$ and

$$\chi(e_{11})((\chi(a')[x, y] + [x, y]\chi(b'))^n - [x, y]) = 0 \tag{4}$$

for all $x, y \in R$. Notice that $\varphi(e_{11}) = e_{11} + e_{21}$ and $\chi(e_{11}) = e_{11} - e_{21}$. Hence left multiplying in the relations (3) and (4) by e_{11}, we get

$$e_{11}((\varphi(a')[x, y] + [x, y]\varphi(b'))^n - [x, y]) = 0 \tag{5}$$

for all $x, y \in R$ and

$$e_{11}((\chi(a')[x,y] + [x,y]\chi(b'))^n - [x,y]) = 0 \tag{6}$$

for all $x, y \in R$.

Then, by the same argument as above, we have $\varphi(b')_{21} = 0 = \chi(b')_{21}$. This gives $b'_{11} - b'_{22} - b'_{12} = 0$ and $-b'_{11} + b'_{22} - b'_{12} = 0$. Both of these imply $b'_{12} = 0$ and $b'_{11} = b'_{22}$, that is $b' = tbt^{-1}$ is central. Hence b must be central. Therefore, (1) reduces to

$$p((c[x,y])^n - [x,y]) = 0 \tag{7}$$

for all $x, y \in R$, where $c = a + b$. Moreover, R is a dense ring of K-linear transformations over a vector space K^2.

Assume there exists $v \neq 0$, such that $\{v, cv\}$ is linear K-independent. By the density of R, there exist $r_1, r_2 \in R$ such that

$$r_1 v = 0; \quad r_1(cv) = v; \quad r_2 v = -v; \quad r_2 cv = 0.$$

Hence

$$[r_1, r_2]v = 0; \quad [r_1, r_2]cv = v; \quad \left(c[r_1, r_2]\right)^n cv = cv.$$

Thus we have

$$0 = \{p((c[r_1, r_2])^n - [r_1, r_2])c\}v = p(c-1)v.$$

Of course for any $u \in V$, $\{u, v\}$ linearly K-dependent implies $p(c-1)u = 0$. If $p(c-1) = 0$, conclusion is obtained. Suppose $p(c-1) \neq 0$. Then there exists $w \in V$ such that $p(c-1)w \neq 0$ and so $\{w, v\}$ are linearly K-independent. Also $p(c-1)(w+v) = p(c-1)w \neq 0$ and $p(c-1)(w-v) = p(c-1)w \neq 0$. By the above argument, it follows that w and cw are linearly K-dependent, as are $\{w+v, c(w+v)\}$ and $\{w-v, c(w-v)\}$. Therefore there exist $\alpha_w, \alpha_{w+v}, \alpha_{w-v} \in K$ such that

$$cw = \alpha_w w, \quad c(w+v) = \alpha_{w+v}(w+v), \quad c(w-v) = \alpha_{w-v}(w-v).$$

In other words we have

$$\alpha_w w + cv = \alpha_{w+v} w + \alpha_{w+v} v \tag{8}$$

and

$$\alpha_w w - cv = \alpha_{w-v} w - \alpha_{w-v} v. \tag{9}$$

By comparing (8) with (9) we get both

$$(2\alpha_w - \alpha_{w+v} - \alpha_{w-v})w + (\alpha_{w-v} - \alpha_{w+v})v = 0 \tag{10}$$

and

$$2cv = (\alpha_{w+v} - \alpha_{w-v})w + (\alpha_{w+v} + \alpha_{w-v})v. \tag{11}$$

By (10), and since $\{w, v\}$ are K-independent and $char(K) \neq 2$, we have $\alpha_w = \alpha_{w+v} = \alpha_{w-v}$. Thus by (11) it follows $2cv = 2\alpha_w v$. This leads a contradiction with the fact that $\{v, cv\}$ is linear K-independent.

In light of this, we may assume that for any $v \in V$ there exists a suitable $\alpha_v \in K$ such that $cv = \alpha_v v$, and standard argument shows that there is $\alpha \in K$ such that $cv = \alpha v$ for all $v \in V$. Hence $(c - \alpha)V = 0$. Therefore, $c = \alpha \in Z(R)$.

Thus our identity (7) reduces to

$$p(c^n[x, y]^n - [x, y]) = 0 \tag{12}$$

for all $x, y \in R$. Now assuming $x = e_{12}$ and $y = e_{22}$, we have $0 = pe_{12}$ which implies $p_{11} = p_{21} = 0$. Similarly assuming $x = e_{21}$ and $y = e_{11}$, we can prove that $p_{22} = p_{12} = 0$, that is $p = 0$, contradiction.

Lemma 2.2. *Let R be a prime ring with extended centroid C and $a, b, p \in R$. If $p \neq 0$ such that $p((a[x, y] + [x, y]b)^n - [x, y]) = 0$ for all $x, y \in R$, where $n \geq 2$ a fixed integer, then R satisfies a nontrivial generalized polynomial identity (GPI).*

Proof. Assume that R does not satisfy any nontrivial GPI. Let $T = U *_C C\{X, Y\}$, the free product of U and $C\{X, Y\}$, the free C-algebra in noncommuting indeterminates X and Y. If R is commutative, then R satisfies trivially a nontrivial GPI, a contradiction. So, R must be noncommutative.

Then, since $p((a[x, y] + [x, y]b)^n - [x, y]) = 0$ is a GPI for R, we see that

$$p((a[X, Y] + [X, Y]b)^n - [X, Y]) = 0 \tag{13}$$

in $T = U *_C C\{X, Y\}$. If $b \notin C$, then b and 1 are linearly independent over C. Thus, (13) implies

$$p(a[X, Y] + [X, Y]b)^{n-1}([X, Y]b) = 0 \tag{14}$$

in T and then by the same argument, $p([X, Y]b)^n = 0$ in T, implying $b = 0$, since $p \neq 0$, a contradiction. Therefore, we conclude that $b \in C$ and hence (13) reduces to

$$p(((a + b)[X, Y])^n - [X, Y]) = 0 \tag{15}$$

that is

$$p(((a + b)[X, Y])^{n-1}(a + b) - 1)[X, Y] = 0 \tag{16}$$

in T. If $a+b \notin C$, then (16) reduces to

$$p((a+b)[X,Y])^{n-1}(a+b)[X,Y]=0 \tag{17}$$

that is $p((a+b)[X,Y])^n = 0$ in T. Since $n \geq 2$, this implies that $a+b=0$, a contradiction. Hence we have $a+b \in C$. Thus the identity (13) becomes that

$$p((a+b)^n[X,Y]^n - [X,Y]) = 0 \tag{18}$$

in T. Since $p \neq 0$, we have that $(a+b)^n[X,Y]^n - [X,Y] = 0$ in T that is R satisfies a nontrivial GPI, a contradiction.

Lemma 2.3. *Let R be a prime ring with extended centroid C and $a, b, p \in R$. Suppose that $p \neq 0$ such that $p((a[x,y]+[x,y]b)^n - [x,y]) = 0$ for all $x, y \in R$, where $n \geq 2$ is a fixed integer. Then one of the following holds:*

(1) *R is commutative;*

(2) *char $(R) \neq 2$, $R \subseteq M_2(C)$, $b \in Z(R)$ with $p(a+b-1)=0$ (In this case n is odd integer);*

(3) *char $(R) = 2$, $R \subseteq M_2(C)$ and $p((a+b)^n - 1) = 0$.*

Proof. We have that R satisfies generalized polynomial identity

$$f(x,y) = p((a[x,y]+[x,y]b)^n - [x,y]) = 0. \tag{19}$$

By Lemma 2.2, we obtain that R satisfies a nontrivial GPI. Since R and U satisfy the same generalized polynomial identities (see [3]), U satisfies $f(x,y)$. In case C is infinite, we have $f(x_1,x_2) = 0$ for all $x, y \in U \otimes_C \overline{C}$, where $\overline{C}$ is the algebraic closure of C. Moreover, both U and $U \otimes_C \overline{C}$ are prime and centrally closed algebras [8]. Hence, replacing R by U or $U \otimes_C \overline{C}$ according to C finite or infinite, without loss of generality we may assume that $C = Z(R)$ and R is a centrally closed C-algebra. By Martindale's theorem [15], R is then a primitive ring having nonzero socle $soc(R)$ with C as the associated division ring. Hence, by Jacobson's theorem [11, p.75], R is isomorphic to a dense ring of linear transformations of a vector space V over C. If $\dim_C V = 1$, then R is commutative, as desired. If $\dim_C V = 2$, then $R \subseteq M_2(C)$. This case gives conclusion (2) and (3) by Lemma 2.1. Thus we consider the case $\dim_C V \geq 3$, and we show that this leads a number of contradictions.

Suppose that there exists some $v \in V$ such that v and bv are linearly C-independent. Since $\dim_C V \geq 3$, we choose another $w' \in V$ such that $\{v, bv, w'\}$ is a linearly C-independent set of vectors. By density, there exist $x, y \in R$ such that

$$xv = 0, \quad xbv = v, \quad xw' = (b-a)v, \quad yv = bv, \quad ybv = w', \quad yw' = 0.$$

Then $0 = p((a[x,y]+[x,y]b)^n - [x,y])v = -pv$.

This implies that if $pv \neq 0$, then by contradiction we may conclude that v and bv are linearly C-dependent. Now choose $v \in V$ such that v and bv are linearly C-independent. Set $W = Span_C\{v, bv\}$. Then $pv = 0$. Since $p \neq 0$, there exists $w \in V$ such that $pw \neq 0$ and then $p(v - w) = -pw \neq 0$. By the previous argument we have that w, bw are linearly C-dependent and $(v - w), b(v - w)$ too. Thus there exist $\alpha, \beta \in C$ such that $bw = \alpha w$ and $b(v - w) = \beta(v - w)$. Then $bv = \beta(v - w) + bw = \beta(v - w) + \alpha w$ i.e., $(\alpha - \beta)w = bv - \beta v \in W$. Now $\alpha = \beta$ implies that $bv = \beta v$, a contradiction. Hence $\alpha \neq \beta$ and so $w \in W$. Again, if $u \in V$ with $pu = 0$ then $p(w + u) \neq 0$. So, $w + u \in W$ forcing $u \in W$. Thus it is observed that $w \in V$ with $pw \neq 0$ implies $w \in W$ and $u \in V$ with $pu = 0$ implies $u \in W$. This implies that $V = W$ i.e., $\dim_C V = 2$, a contradiction.

Hence, in any case, v and bv are linearly C-dependent for all $v \in V$. Then by standard arguments, it follows that $b \in C$.

Therefore, from (19) we have that R satisfies generalized polynomial identity

$$f(x_1, x_2) = p((a'[x, y])^n - [x, y]), \tag{20}$$

where $a' = a + b$. Now if v and $a'v$ are linearly C-independent for some $v \in V$, then there exists $w \in V$ such that $\{v, a'v, w\}$ forms a set of linearly C-independent set of vectors, since $\dim_C V \geq 3$. Then again by density, there exist $x, y \in R$ such that

$$xv = 0, \quad xa'v = v, \quad xw = a'v; yv = a'v, \quad ya'v = w, \quad yw = 0.$$

In this case we get $0 = p((a'[x, y])^n - [x, y])v = -pv$. Since $p \neq 0$, by the same argument as above, this leads a contradiction. Hence, by above argument we conclude $a' \in C$. Therefore, the identity (20) reduces to

$$p(a'^n[x, y]^n - [x, y]) = 0 \tag{21}$$

for all $x, y \in R$.

Now let $\dim_C V = k$. Then $k \geq 3$ and $R \cong M_k(C)$. Replacing $x = e_{ii}$ and $y = e_{ij}$ in (21), we get that $-pe_{ij} = 0$. This implies $p = 0$, a contradiction.

Theorem 2.4. *Let R be a noncommutative prime ring with its Utumi ring of quotients U, $C = Z(U)$ the extended centroid of R, F a generalized derivation of R and I a nonzero ideal of R. Suppose that there exists $0 \neq a \in R$ such that $a(F([x, y])^n - [x, y]) = 0$ for all $x, y \in I$, where $n \geq 2$ is a fixed integer. Then one of the following holds:*

(1) *char $(R) \neq 2$, $R \subseteq M_2(C)$, $F(x) = bx$ for all $x \in R$ with $a(b - 1) = 0$ (In this case n is an odd integer);*

(2) *char $(R) = 2$, $R \subseteq M_2(C)$ and $F(x) = bx + [c, x]$ for all $x \in R$ with $a(b^n - 1) = 0$.*

Proof. By our assumption we have,

$$a(F([x,y])^n - [x,y]) = 0$$

for all $x, y \in I$.

Since I, R and U satisfy the same generalized polynomial identities (see [3]) as well as the same differential identities (see [14]), they also satisfy the same generalized differential identities by Remark. Hence,

$$a(F([x,y])^n - [x,y]) = 0$$

for all $x, y \in U$, where $F(x) = bx + d(x)$, for some $b \in U$ and derivations d of U. Hence, U satisfies

$$a((b[x,y] + d([x,y]))^n - [x,y]) = 0. \tag{22}$$

Now we divide the proof into two cases:

Let $d(x) = [c, x]$ for all $x \in U$ i.e., d is an inner derivation of U. Then from (22), we obtain that U satisfies

$$a(((b+c)[x,y] - [x,y]c)^n - [x,y]) = 0. \tag{23}$$

By Lemma 2.3, since $a \neq 0$ and R is noncommutative, one of the following holds:

(1) char $(R) \neq 2$, $R \subseteq M_2(C)$, $c \in Z(R)$ with $a(b-1) = 0$. In this case n is odd integer and $F(x) = bx$ for all $x \in R$.

(2) char $(R) = 2$, $R \subseteq M_2(C)$ and $a(b^n - 1) = 0$. In this case $F(x) = bx + [c, x]$ for all $x \in R$.

Next assume that d is not an inner derivation of U. Then by Kharchenko's theorem [12], we have that U satisfies

$$a((b[x,y] + [s,y] + [x,t])^n - [x,y]) = 0. \tag{24}$$

In particular, for $y = 0$, we have that U satisfies

$$a[x,t]^n = 0. \tag{25}$$

Let $w = [x,y]^n$. Then $aw = 0$. From (25), we can write $a[p, wqa]^n = 0$ for all $p, q \in U$. Since $aw = 0$, it reduces to $a(pwqa)^n = 0$. This can be written as $(wqap)^{n+1} = 0$ for all $p, q \in R$. By Levitzki's lemma [10, Lemma 1.1], $wqa = 0$ for all $q \in U$. Since U is prime and $a \neq 0$, we have $w = 0$. Thus $w = [x,y]^n = 0$ for all $x, y \in U$. Then by Herstein [9, Theorem 2], U and so R is commutative, contradicting.

Corollary 2.5. *Let R be a prime ring with C the extended centroid of R, d a derivation of R and I a nonzero ideal of R. Suppose that there exists $0 \neq a \in R$ such that $a(d([x,y])^n - [x,y]) = 0$ for all $x, y \in I$, where $n \geq 1$ is a fixed integer. Then R must be commutative.*

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