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INSERTION-OF-FACTORS-PROPERTY SKEWED BY RING ENDOMORPHISMS

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Abstract. In this paper, we investigate the Insertion-of-Factors-Property (simply IFP), (quasi-)Baer property, and Armendariz property on skew power series (polynomial) rings and introduce the concept of (strongly) σ-skew IFP and extend many of related basic results to the wider classes. When a ring R has σ-skew IFP and σ is a monomorphism of R we prove that R is Baer if and only if R is quasi-Baer if and only if $R[[x;\sigma]]$ ($R[x;\sigma]$) is Baer if and only if $R[[x;\sigma]]$ ($R[x;\sigma]$) is quasi-Baer. We also prove that if R is a skew power-serieswise σ-Armendariz ring then R has strongly σ-skew IFP and $R[[x;\sigma]]$ has IFP. Several known results follow as consequences of our results. In particular, we provide a σ-skew power-serieswise Armendariz ring but does not have IFP.

1. INTRODUCTION

Throughout this paper, all rings are associative with identity. We use $R[x]$ ($R[[x]]$) to denote the polynomial ring (the power series ring) with an indeterminate x over R. For any polynomial $f(x)$ in $R[x]$ ($R[[x]]$), let $C_{f(x)}$ denote the set of all coefficients of $f(x)$. Let $\mathbb{Z}$ and $\mathbb{Z}_n$ denote the ring of integers and the ring of integers modulo n, respectively.

Due to Bell [3], a ring R is called to satisfy the *Insertion-of-Factors-Property* (simply, *IFP*) if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Narbonne [20] and Shin [24] used the terms *semicommutative* and *SI* for the IFP, respectively. Commutative rings clearly have IFP, and any reduced ring (i.e., a ring without nonzero nilpotent elements) has IFP by a simple computation. There exist many non-reduced commutative rings (e.g., $\mathbb{Z}_{n^l}$ for $n, l \geq 2$), and many noncommutative reduced rings (e.g., direct products

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of noncommutative domains). A ring is called *Abelian* if every idempotent is central. Rings that have IFP are Abelian by a simple computation.

Following Başer et al. [2, Definition 2.1], an endomorphism σ of a ring R is called to have *skew IFP* if whenever $ab = 0$ for $a, b \in R$, $aR\sigma(b) = 0$, and a ring R is called to have *σ-skew IFP* if there exists an endomorphism σ having skew IFP of R. A ring R has σ-skew IFP if and only if for $a, b \in R$, $ab = 0$ implies $aR\sigma^n(b) = 0$ for all $n \geq 1$ by [2, Remark 2.2]. According to Krempa [18], an endomorphism σ of a ring R is called *rigid* if $a\sigma(a) = 0$ implies $a = 0$ for $a \in R$, and a ring R is called *σ-rigid* if there exists a rigid endomorphism σ of R. Note that any rigid endomorphism of a ring is a monomorphism and σ-rigid rings are reduced by [8, Proposition 5]. Notice that a ring R is σ-rigid if and only if R is reduced and has σ-skew IFP and σ is a monomorphism by [2, Theorem 2.4].

Rege and Chhawchharia [23] called a ring R *Armendariz* if whenever any polynomials $f(x), g(x) \in R[x]$ satisfy $f(x)g(x) = 0$, $ab = 0$ for all $a \in C_{f(x)}$ and $b \in C_{g(x)}$. This nomenclature was used by them since it was Armendariz [1, Lemma 1] who initially showed that a reduced ring always satisfies this condition. For a semiprime right Goldie ring R, R is Armendariz if and only if R has IFP. However, the IFP ring property and the Armendariz ring property don't imply each other by [23, Example 3.2] and [13, Example 14]. On the other hand, a ring R is called *power-serieswise Armendariz* [15] if $ab = 0$ for all $a \in C_{f(x)}$ and $b \in C_{g(x)}$ whenever $f(x), g(x) \in R[[x]]$ satisfy $f(x)g(x) = 0$. Every power-serieswise Armendariz ring is obviously Armendariz by definition, but the converse does not hold by [15, Example 2.1]. Reduced rings are power-serieswise Armendariz by [15, Lemma 2.3(1)].

For a ring R with an endomorphism σ, a *skew polynomial* ring (also called an *Ore extension of endomorphism type*) $R[x;\sigma]$ of R is the ring obtained by giving the polynomial ring over R with the new multiplication $xr = \sigma(r)x$ for all $r \in R$, while $R[[x;\sigma]]$ is called a *skew power series ring*. Note that $\sigma(1) = 1$ for any skew power series (skew polynomial) ring $R[[x;\sigma]](R[x;\sigma])$, since $1x^n = x^n = x1x^{n-1} = \sigma(1)x^n$ for any $n \geq 1$ where 1 is the identity of R.

Following [2], a ring R is called *skew power-serieswise σ-Armendariz* if $a_ib_j = 0$ for all i and j whenever $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} a_ix^i, q(x) = \sum_{j=0}^{\infty} b_jx^j \in R[[x;\sigma]]$. Skew power-serieswise σ-Armendariz rings are skew Armendariz in the sense of [21], and σ-rigid rings are skew power-serieswise σ-Armendariz by [8, Proposition 17], but there exist many examples of non-reduced (and hence non-σ-rigid) skew power-serieswise σ-Armendariz rings as we can see in [22, Section 4]. It is obvious that skew power-serieswise σ-Armendariz property of a ring is inherited to its subrings, and that every skew power-serieswise σ-Armendariz ring has IFP. If R is a skew power-serieswise σ-Armendariz ring, then σ is clearly a monomorphism by help of [12, Proposition 1.3]. Hong et al. [11] called a ring R *σ-skew power-serieswise Armendariz* if whenever $p(x) = \sum_{i=0}^{\infty} a_ix^i, q(x) = \sum_{j=0}^{\infty} b_jx^j$ in $R[[x;\sigma]]$ satisfy

$p(x)q(x) = 0$, then $a_i\sigma^i(b_j) = 0$ for all i, j. It is shown that every σ-skew power-serieswise Armendariz ring has σ-skew IFP by [11, Lemma 3.1(2)].

Based on the above, in this paper, we investigate the Insertion-of-Factors-Property skewed by ring endomorphisms, (quasi-)Baer property, and Armendariz property on skew power series (polynomial) rings and so several known results follow as consequences of our results. We change over from "a σ-semicommutative ring" and "a skew power series Armendariz ring with the endomorphism σ" (in [2]) to "a ring has σ-skew IFP" and "a skew power-serieswise σ-Armendariz ring" respectively, so as to cohere with other related definitions.

From now on, σ and id_R denote a nonzero endomorphism and the identity endomorphism of the ring R respectively, unless specified otherwise.

2. Skew Power Series Rings over Baer and P.P.-Rings

In this section, we first show that (quasi-)Baer property can be extended from R to $R[[x;\sigma]]$ ($R[x;\sigma]$), and then define the concept of strongly σ-skew IFP and extend many of related basic results to the wider classes.

A ring R is called *Baer* by Kaplansky [14] if the right annihilator of every nonempty subset of R is generated by an idempotent. An example of Cohn shows that the n by n full matrix ring $Mat_n(\mathbb{Z})$ is Baer but $Mat_n(\mathbb{Z})[x]$ and $Mat_n(\mathbb{Z})[[x]]$ are not (see [1]). For an Armendariz ring R, R is Baer if and only if $R[x]$ is Baer ([1] and [16]).

According to Clark [4], a ring R is called *quasi-Baer* if the right annihilator of each right ideal of R is generated (as a right ideal) by an idempotent. It is well-known that these two concepts are left-right symmetric. A ring R is called a *right* (*left*) *p.p.*-ring if the right (left) annihilator of an element of R is generated by an idempotent. R is called a *p.p.*-ring if it is both a left and right p.p.-ring.

Lemma 2.1. *Let a ring R have σ-skew IFP. Then we have the following results.*

(1) $\sigma(1) = \sigma^n(1)$ *for all* $n \geq 2$.

(2) *Let* $e^2 = e \in R$. *Then* $\sigma(e) = e\sigma(e) = e\sigma(1) = e\sigma^n(e)$ *for all* $n \geq 2$.

(3) *If σ is a monomorphism, then* $\sigma(1) = 1$, *and hence* $\sigma(e) = e$ *for any* $e^2 = e \in R$.

(4) *If σ is an epimorphism, then* $\sigma(e) = e$ *for every* $e^2 = e \in R$.

Proof. (1) From $\sigma(1)(1-\sigma(1)) = 0$, we get $0 = \sigma(1)\sigma(1-\sigma(1)) = \sigma(1)(\sigma(1) - \sigma^2(1)) = \sigma(1) - \sigma^2(1)$ since R has σ-skew IFP. This yields $\sigma(1) = \sigma^2(1)$, so $\sigma(1) = \sigma^2(1) = \sigma(\sigma(1)) = \sigma(\sigma^2(1)) = \sigma^3(1) = \cdots = \sigma^n(1)$ for all $n \geq 2$.

(2) Let $e^2 = e \in R$ and $n \geq 2$. From $e(1-e) = 0$ and $(1-e)e = 0$, we get $0 = e\sigma(1-e) = e(\sigma(1)-\sigma(e)) = e\sigma(1) - e\sigma(e)$ and $0 = (1-e)\sigma(e) = \sigma(e) - e\sigma(e)$ since R is σ-skew IFP. So $\sigma(e) = e\sigma(e) = e\sigma(1)$. Moreover $0 = e\sigma^n(1-e) = e(\sigma^n(1) - \sigma^n(e)) = e\sigma(1) - e\sigma^n(e)$ (hence $\sigma(e) = e\sigma(1) = e\sigma^n(e)$) by (1).

(3) From (1), we get $\sigma(1-\sigma(1)) = 0$ and so $\sigma(1) = 1$ since σ is a monomorphism. The fact that $\sigma(e) = e$ comes from (2).

(4) This follows from (2), since any epimorphism σ always satisfies $\sigma(1) = 1$. ■

Observe that if a ring R has σ-skew IFP and σ is a monomorphism of R, then $\sigma(e) = e$ for $e^2 = e \in R$ and the set of all idempotents in $R[[x;\sigma]]$ coincides with the set of all idempotents of R and $R[[x;\sigma]]$ is Abelian by Lemma 2.1(3) and [2, Proposition 2.6 and Proposition 3.9], and hence $ae = 0$ for $a, e^2 = e \in R$ implies that $a(x^s R x^t)e = 0$ for any $s, t \geq 0$. We will use these freely in the proofs of the next three results.

For a nonempty subset A of a ring R, we write $r_R(A) = \{s \in R \mid as = 0$ for any $a \in A\}$ which is called the right annihilator of A in R. Left annihilators are denoted similarly, written by $\ell_R(A)$.

Theorem 2.2. *Let a ring R have σ-skew IFP and σ a monomorphism of R. Then the following conditions are equivalent:*

(1) *R is Baer.*

(2) *R is quasi-Baer.*

(3) *$R[[x;\sigma]]$ ($R[x;\sigma]$) is quasi-Baer.*

(4) *$R[[x;\sigma]]$ ($R[x;\sigma]$) is Baer.*

Proof. We only show that (2)⇒(1), (1)⇒(4), and (3)⇒(2).

(2)⇒(1): Let A be a nonempty subset of R. If R is quasi-Baer, then $r_R(AR) = eR$ and so $eR \subseteq r_R(A)$. Now, let $c \in r_R(A)$. Then $Ac = 0$. Since R is σ-skew IFP, $AR\sigma(c) = 0$ and hence $\sigma(c) \in r_R(AR) = eR$. But $\sigma(c) = e\sigma(c) = \sigma(ec)$, and so $c = ec \in eR$ since σ is a monomorphism. This yields $r_R(A) = eR$ and therefore R is Baer.

(1)⇒(4): Suppose that e and f are idempotents in R such that $\ell_R(A) = Re$ and $\ell_R(A_1) = Rf$ for some subset A of R, where $A_1 = \{\sigma(a) \mid a \in A\}$, i.e., $A_1 = \sigma(A)$. By hypothesis, $eA = 0$ implies $eA_1 = 0$, so $e \in Rf$ and $e = ef = fe$. Also, $0 = fA_1 = \sigma(f)\sigma(A) = \sigma(fA)$, and so $fA = 0$ since σ is a monomorphism. This yields $f \in \ell_R(A)$ and $f = fe = ef$. Thus $e = f$ and so

$$\ell_R(A) = Re = Rf = \ell_R(A_1) = \ell_R(\sigma(A)),$$

entailing that $\ell_R(A) = Re = \ell_R(\sigma^n(A))$ for all $n \geq 1$.

Let R be a Baer ring. Let $T = R[[x;\sigma]]$, S be a subset of T, and $I = ST$. Note $\ell_T(S) = \ell_T(I)$. If $I = 0$ then we are done. So assume $I \neq 0$ and

$$I_0 = \{a \in R \mid a \text{ is the nonzero coefficient of the term of lowest degree of } p(x) \text{ when } 0 \neq p(x) \in I\}.$$

Since R is Baer, $\ell_R(I_0) = Re$ for some $e^2 = e \in R$. Then $\ell_R(I_0) = \ell_R(I_n)$ for all $n \geq 1$ by the result above, where $I_n = \sigma^n(I_0)$. Let $q(x) = \sum_{i=0}^{\infty} a_i x^i \in I$. Then $ea_0 = a_0 e = 0$ whether $a_0 = 0$ or not. Note $eq(x) = \sum_{i=0}^{\infty} ea_i x^i = ea_1 x + ea_2 x^2 + \cdots = q(x)e \in I$. If $ea_1 = a_1 e \neq 0$ then $ea_1 \in I_0$ and this yields $ea_1 = e(ea_1) = e(a_1 e) = 0$, a contradiction. So $ea_1 = 0$, and we obtain inductively that $ea_2 = ea_3 = \cdots = 0$, entailing $eq(x) = 0$. This yields $e \in \ell_T(I)$ and $Te \subseteq \ell_T(I) = \ell_T(S)$.

Conversely let $r(x) = \sum_{i=0}^{\infty} b_i x^i \in \ell_T(I)$ and we shall show $r(x) = r(x)e$. Let $0 \neq p(x) = c_0 x^j + c_1 x^{j+1} + c_2 x^{j+2} + \cdots$ be any in I with $c_0 \neq 0$. Note that $0 \neq \sigma^n(c_0)$ is any in $I_n = \sigma^n(I_0)$. Then $r(x)p(x) = 0$ and so $b_0 c_0 = 0$, entailing $b_0 \in \ell_R(I_0) = Re \subseteq \ell_T(I)$. Thus $b_0 = b_0 e$ and $b_0 p(x) = b_0 ep(x) = 0$, and so $b_0 c_k = 0$ for all $k = 1, 2, \ldots$. Then we have $(b_1 x + b_2 x^2 + \cdots + \cdots)p(x) = 0$ and so $b_1 \sigma(c_0) = 0$. This yields $b_1 \in \ell_R(I_1) = \ell_R(I_0) = Re \subseteq \ell_T(I)$, so $b_1 = b_1 e$. Then $(b_1 x)p(x) = (b_1 ex)p(x) = (b_1 x)ep(x) = 0$, and so $b_1 \sigma(c_k) = 0$ for all $k = 1, 2, \ldots$. Consequently we have $(b_2 x^2 + b_3 x^3 + \cdots)p(x) = 0$, entailing $b_2 \sigma^2(c_0) = 0$. This also yields $b_2 \in \ell_R(I_2) = \ell_R(I_0) = Re \subseteq \ell_T(I)$, so $b_2 = b_2 e$. From $(b_2 x^2)p(x) = (b_2 ex^2)p(x) = (b_2 x^2)ep(x) = 0$, we also get $b_2 \sigma^2(c_k) = 0$ for all $k = 1, 2, \ldots$. Proceeding in this manner, we inductively obtain $b_i = b_i e$ for all $i \geq 0$, and so $r(x) = r(x)e \in Te$. Hence $\ell_T(I) \subseteq Te$ and thus $\ell_T(I) = Te$.

(3)$\Rightarrow$(2): Suppose that $R[[x;\sigma]]$ is quasi-Baer. Let I be a right ideal of R. Then by hypothesis, $r_{R[[x;\sigma]]}(I[[x;\sigma]]) = eR[[x;\sigma]]$ for some $e^2 = e \in R$. Thus $Ie = 0$ and so $eR \subseteq r_R(I)$. To prove the reverse inclusion, let $a \in r_R(I)$. Then $IRa = 0$ and $IRx^t a = 0$ for any integer $t \geq 1$ because R has σ-skew IFP. So $IR[[x;\sigma]]a = 0$, and hence $a \in r_{R[[x;\sigma]]}(I[[x;\sigma]]) = eR[[x;\sigma]]$. Therefore $a \in eR$, proving that $r_R(I) \subseteq eR$.

The preceding proof is also available to the skew polynomial ring case. ∎

Proposition 2.3. *Let a ring R have σ-skew IFP and σ a monomorphism of R. If $R[[x;\sigma]]$ ($R[x;\sigma]$) is a right p.p.-ring then R is a right p.p.-ring.*

Proof. Suppose that $R[[x;\sigma]]$ is right p.p.. Let $a \in R$. Then $r_{R[[x;\sigma]]}(a) = eR[[x;\sigma]]$ for some $e^2 = e \in R$ by hypothesis. Thus $eR \subseteq r_R(a)$. By the almost same argument as in the proof (2)$\Rightarrow$(1) of Theorem 2.2, we obtain $r_R(a) \subseteq eR$, and consequently $r_R(a) = eR$.

The preceding proof is also available to the skew polynomial ring case. ∎

Notice that the converse of Proposition 2.3 does not hold by help of [8, p.225]. However, we have the following result.

Proposition 2.4. *Let a ring R have σ-skew IFP and σ a monomorphism of R. Then the following conditions are equivalent:*

(1) *R is a right p.p.-ring.*

(2) *For any $f(x) = \sum_{i=0}^{n} a_i x_i \in R[[x;\sigma]]$, $r_{R[[x;\sigma]]}(f(x)) = eR[[x;\sigma]]$ for some $e^2 = e \in R[[x;\sigma]]$.*

(3) *For any* $f(x) = \sum_{i=0}^{n} a_i x^i \in R[[x;\sigma]]$, $r_{R[[x;\sigma]]}(f(x)R[[x;\sigma]]) = eR[[x;\sigma]]$ *for some* $e^2 = e \in R[[x;\sigma]]$.

Proof. (1)⇒(2): Assume that R is right p.p.. We first note that R is σ-rigid. For, if $a\sigma(a) = 0$ for $a \in R$ then $\sigma(a)\sigma(a) = 0$ and so $a = 0$ since R is reduced. Let $f(x) = \sum_{i=0}^{n} a_i x^i \in R[[x;\sigma]]$. By hypothesis, $r_R(a_i) = e_i R$ for each i and some $e_i^2 = e_i \in R$. Putting $e = e_i e_2 \cdots e_n$ then $e^2 = e \in R$. Thus $a_i e = 0$ for any $0 \leq i \leq n$. Then $a_i(x^s)e = 0$ for any $s \geq 0$ and i, and so $f(x)e = 0$. Thus $eR[[x;\sigma]] \subseteq r_{R[[x;\sigma]]}(f(x))$. For the reverse inclusion, let $q(x) = \sum_{j=0}^{\infty} b_j x^j \in r_{R[[x;\sigma]]}(f(x))$. Then $f(x)q(x) = 0$ and so $a_i b_j = 0$ for all i and j by [8, Proposition 17], since R is σ-rigid. This implies $b_j \in r_R(a_i R) = e_i R$, and thus $b_j = e_i b_j$ for all j and $i = 0, 1, \ldots, n$. Consequently, we get $b_j = eb_j$ for all j, proving that $q(x) = eq(x) \in eR[[x;\sigma]]$.

(2)⇒(3): Assume (2). Let $f(x) = \sum_{i=0}^{n} a_i x^i \in R[[x;\sigma]]$. Then $r_{R[[x;\sigma]]}(f(x)R[[x;\sigma]]) \subseteq r_{R[[x;\sigma]]}(f(x)) = eR[[x;\sigma]]$ for some $e^2 = e \in R$. Moreover, we get $f(x)e = 0$ and so $a_i(x^s R x^t)e = 0$ for any $s, t \geq 0$ and all i. This yields $f(x)R[[x;\sigma]]e = 0$ and therefore $eR[[x;\sigma]] \subseteq r_{R[[x;\sigma]]}(aR[[x;\sigma]])$.

(3)⇒(1): Assume (3). Let $a \in R$. We first show that $r_R(aR) = eR$ for some $e^2 = e \in R$. By assumption, we have $r_{R[[x;\sigma]]}(aR[[x;\sigma]]) = eR[[x;\sigma]]$ for some $e^2 = e \in R$. Then we get $aRe = 0$, and so $eR \subseteq r_R(aR)$. If $b \in r_R(aR)$, then $aRb = 0$ and so $aR[[x;\sigma]]b = 0$ since R has σ-skew IFP. Thus $b \in eR[[x;\sigma]]$ and so $b \in eR$, showing that $r_R(aR) = eR$ for any $a \in R$. From this result, we have $eR \subseteq r_R(a)$. We now let $b \in r_R(a)$. Since R is σ-skew IFP, $aR\sigma(b) = 0$ and hence $\sigma(b) \in r_R(aR) = eR$. By the same argument as in the proof (2)⇒(1) of Theorem 2.2, we have $b \in eR$ and thus R is right p.p.. ■

An endomorphism σ of a ring R is called to have *strongly skew IFP* if $aR\sigma^n(b) = 0$ for all $n \geq 0$ whenever $ab = 0$ for $a, b \in R$, and a ring R is called to have *strongly σ-skew IFP* if there exists an endomorphism σ having strongly skew IFP of R. Every domain with any endomorphism σ clearly has strongly σ-skew IFP, and every ring having strongly σ-skew IFP has σ-skew IFP but not conversely by [2, Example 2.7]. It can be easily checked that a ring R has strongly σ-skew IFP when $R[[x;\sigma]]$ has IFP. However, there exists a ring R having IFP with an endomorphism σ which does not have strongly σ-skew IFP. For example, the ring $R = D \oplus D$ with $\sigma((a, b)) = (b, a)$, where D is any reduced ring.

Lemma 2.5. *Let R be a ring with an endomorphism σ. Then the following conditions are equivalent:*

(1) *R has strongly σ-skew IFP.*

(2) *$aR[[x;\sigma]]b = 0$ whenever $ab = 0$ for $a, b \in R$.*

(3) *$aR[x;\sigma]b = 0$ whenever $ab = 0$ for $a, b \in R$.*

(4) *R has IFP and σ-skew IFP.*

Proof. It suffices to show (1)⇒(2). Assume (1). Let $ab = 0$ for $a, b \in R$. Then $aR\sigma^n(b) = 0$ and so $a(Rx^n)b = 0$ for all $n \geq 0$. This yields $aR[[x;\sigma]]b = 0$. ■

For an algebra R (with or without identity) over a nonzero commutative ring S, the *Dorroh extension* of R by S is the ring $D = R \times S$ with operations $(r_1, s_1) + (r_2, s_2) = (r_1 + r_2, s_1 + s_2)$ and $(r_1, s_1)(r_2, s_2) = (r_1r_2 + s_1r_2 + s_2r_1, s_1s_2)$, where $r_i \in R$ and $s_i \in S$. We only consider the case of with identity in this note. For an S-endomorphism σ of R and the Dorroh extension D of R by S, the nonzero map $\bar{\sigma} : D \to D$ defined by $\bar{\sigma}(r, s) = (\sigma(r), s)$ is an S-algebra homomorphism.

Proposition 2.6. *Let S be a domain and σ an S-endomorphism of a ring R with $\sigma(1) = 1$. Then R has (strongly) σ-skew IFP if and only if the Dorroh extension D of R by S has (strongly) $\bar{\sigma}$-skew IFP.*

Proof. It is enough to show the necessity. Note that $s \in S$ is identified with $s1 \in R$ and so $R = \{r + s \mid (r, s) \in D\}$. Suppose that R has strongly σ-skew IFP. Let $(r_1, s_1), (r_2, s_2) \in D$ with $(r_1, s_1)(r_2, s_2) = 0$. Then $r_1r_2 + s_1r_2 + s_2r_1 = 0$ and $s_1s_2 = 0$. Since S is a domain, $s_1 = 0$ or $s_2 = 0$. If $s_1 = 0$, then $0 = r_1r_2 + s_2r_1 = r_1(r_2 + s_2)$. Since R has strongly σ-skew IFP, $0 = r_1a\sigma^n(r_2 + s_21) = r_1a\sigma^n(r_2) + r_1as_2$ for any $a \in R$ and every $n \geq 0$. For any $(r, s) \in D$ and every $n \geq 0$, $(r_1, 0)(r, s)\bar{\sigma}^n((r_2, s_2)) = (r_1r\sigma^n(r_2) + r_1(r + s)s_2, 0) = 0$. If $s_2 = 0$, then we similarly get $r_1a\sigma^n(r_2) + s_1a\sigma^n(r_2) = 0$ for any $a \in R$ and every $n \geq 0$. For any $(r, s) \in D$ and every $n \geq 0$, $(r_1, s_1)(r, s)\bar{\sigma}^n((r_2, 0)) = (r_1(r+s)\sigma^n(r_2) + s_1(r + s)\sigma^n(r_2), 0) = 0$. Consequently, D has strongly $\bar{\sigma}$-skew IFP. ■

Proposition 2.7. *Let R be an Armendariz ring with an endomorphism σ. Then R has (strongly) σ-skew IFP if and only if $R[x]$ has (strongly) $\bar{\sigma}$-skew IFP, where $\bar{\sigma}(\sum_{i=0}^{m} a_ix^i) = \sum_{i=0}^{m} \sigma(a_i)x^i$.*

Proof. It suffices to show the necessity. Assume that R has strongly σ-skew IFP. Let $f(x), g(x) \in R[x]$ with $f(x)g(x) = 0$. Since R is Armendariz and has strongly σ-skew IFP, we get $aR\sigma^n(b) = 0$ for all $a \in C_{f(x)}, b \in C_{g(x)}$ and every $n \geq 0$, and so $f(x)R[x]\bar{\sigma}^n(g(x)) = 0$. Therefore $R[x]$ has strongly $\bar{\sigma}$-skew IFP. ■

The following asserts that the condition "R is an Armendariz ring" in Proposition 2.7 is not superfluous.

Example 2.8. We apply the ring construction and argument in [17, Example 2.1]. Consider the free algebra $A = \mathbb{Z}_2\langle a_0, a_1, a_2, b_0, b_1, b_2, c\rangle$ with noncommuting indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$ over $\mathbb{Z}_2$. Define an automorphism δ of A by

$$a_0, a_1, a_2, b_0, b_1, b_2, c \mapsto b_0, b_1, b_2, a_0, a_1, a_2, c,$$

respectively. Let B be the set of all polynomials with zero constant terms in A and consider the ideal I of A generated by

$$a_0b_0, a_0b_1 + a_1b_0, a_0b_2 + a_1b_1 + a_2b_0, a_1b_2 + a_2b_1, a_2b_2, a_0rb_0, a_2rb_2,$$
$$b_0a_0, b_0a_1 + b_1a_0, b_0a_2 + b_1a_1 + b_2a_0, b_1a_2 + b_2a_1, b_2a_2, b_0ra_0, b_2ra_2,$$
$$(a_0 + a_1 + a_2)r(b_0 + b_1 + b_2), (b_0 + b_1 + b_2)r(a_0 + a_1 + a_2),$$
$$a_0a_0, a_2a_2, a_0ra_0, a_2ra_2, b_0b_0, b_2b_2, b_0rb_0, b_2rb_2, r_1r_2r_3r_4,$$
$$a_0a_1 + a_1a_0, a_0a_2 + a_1a_1 + a_2a_0, a_1a_2 + a_2a_1,$$
$$b_0b_1 + b_1b_0, b_0b_2 + b_1b_1 + b_2b_0, b_1b_2 + b_2b_1,$$
$$(a_0 + a_1 + a_2)r(a_0 + a_1 + a_2), (b_0 + b_1 + b_2)r(b_0 + b_1 + b_2),$$

where $r, r_1, r_2, r_3, r_4 \in B$. Then clearly $B^4 \subseteq I$. Set $R = A/I$. Since $\delta(I) \subseteq I$, we can obtain an automorphism σ of R by defining $\sigma(s + I) = \delta(s) + I$ for $s \in A$. We identify every element of A with its image in R for simplicity. Observe that $(a_0+a_1x^2+a_2x^4)(b_0+b_1x^2+b_2x^4) = 0$ for $a_0+a_1x^2+a_2x^4, b_0+b_1x^2+b_2x^4 \in R[x]$, but $(a_0+a_1x^2+a_2x^4)c\bar{\sigma}(b_0+b_1x^2+b_2x^4) = (a_0+a_1x^2+a_2x^4)c(a_0+a_1x^2+a_2x^4) \neq 0$ since $a_0ca_1 + a_1ca_0 \neq 0$. Thus $R[x]$ does not have $\bar{\sigma}$-skew IFP. Moreover, R is not Armendariz. In fact, $a_0b_1 \neq 0$ with $(a_0 + a_1x^2 + a_2x^4)(b_0 + b_1x^2 + b_2x^4) = 0$.

Next we show that R has strongly σ-skew IFP. A monomial usually means a product of the indeterminates $a_0, a_1, a_2, b_0, b_1, b_2, c$, and a monomial of degree n means a product of exactly n number of indeterminates. Let H_n be the set of all linear combinations of monomials of degree n over $\mathbb{Z}_2$. Notice that H_n is finite for any n and that the ideal I of R is homogeneous (i.e., if $\sum_{i=1}^{s} r_i \in I$ with $r_i \in H_i$ then every r_i is in I).

Claim 1. If $f_1g_1 \in I$ for $f_1, g_1 \in H_1$ then $f_1B\sigma^n(g_1) \subseteq I$ for all $n \geq 0$.

Proof. Suppose that $f_1g_1 \in I$ for $f_1, g_1 \in H_1$. Then, by the definition of I, we have only the following cases: $(f_1 = a_0, g_1 = b_0), (f_1 = a_2, g_1 = b_2), (f_1 = a_0, g_1 = a_0), (f_1 = a_2, g_1 = a_2), (f_1 = b_0, g_1 = a_0), (f_1 = b_2, g_1 = a_2), (f_1 = b_0, g_1 = b_0), (f_1 = b_2, g_1 = b_2), (f_1 = a_0 + a_1 + a_2, g_1 = b_0 + b_1 + b_2), (f_1 = b_0 + b_1 + b_2, g_1 = a_0+a_1+a_2), (f_1 = a_0+a_1+a_2, g_1 = a_0+a_1+a_2), (f_1 = b_0+b_1+b_2, g_1 = b_0+b_1+b_2)$. So we obtain **Claim 1**, using the definition of I again.

Claim 2. If $fg \in I$ for $f, g \in B$ then $fr\sigma^n(g) \in I$ for all $r \in B$ and $n \geq 0$.

Proof. Let $f = f_1 + f_2 + f_3 + f_4, g = g_1 + g_2 + g_3 + g_4$ and $r = r_1 + r_2 + r_3 + r_4$, where $f_1, g_1, r_1 \in H_1$, $f_2, g_2, r_2 \in H_2$, $f_3, g_3, r_3 \in H_3$, and $f_4, g_4, r_4 \in I$. Note that $H_i \subseteq I$ for $i \geq 4$. So $fr\sigma^n(g) = f_1r_1\sigma^n(g_1) + h$ for some $h \in I$. But $fg \in I$ implies $f_1g_1 \in I$ since I is homogeneous; hence $f_1r_1\sigma^n(g_1) \in I$ by Claim 1. Consequently $fr\sigma^n(g) \in I$.

Now let $yz \in I$ for $y, z \in A$. Write $y = \alpha + y', z = \beta + z'$ for some $\alpha, \beta \in \mathbb{Z}_2$

and some $y', z' \in B$. So $\alpha\beta + \alpha z' + y'\beta + y'z' = yz \in I$; hence $\alpha = 0$ or $\beta = 0$. Assume $\alpha = 0$. Then $y'\beta + y'z' \in I$. If $\beta = 1$ then $y' \in I$ because I is homogeneous; hence $yr\sigma^n(z) = y'r\sigma^n(z) \in I$ for all $r \in A$. If $\beta = 0$ then $y'z' \in I$ and so $yr\sigma^n(z) = y'r\sigma^n(z') \in I$ for all $r \in A$ by Claim 2. The proof of the case of $\beta = 0$ is similar. Therefore R has strongly σ-skew IFP.

3. The Armendariz Property on Skew Power Series Rings

In this section, we study the Armendariz property on skew power series rings relating the skew IFP property, and investigate their properties.

Note that every σ-skew power-serieswise Armendariz ring has σ-skew IFP by [11, Lemma 3.1(2)], but the converse is not true in general by [11, Example 3.3].

We first provide an example of a ring that has strongly σ-skew IFP and it is not σ-skew power-serieswise Armendariz as follows.

Given a ring R and an (R, R)-bimodule M, the *trivial extension* of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and the following multiplication: $(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2)$. This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

Example 3.1. Consider the trivial extension $R = T(\mathbb{Z}_4, \mathbb{Z}_4)$ of $\mathbb{Z}_4$. Let $\sigma : R \to R$ be an endomorphism defined by $\sigma((a, b)) = (a, -b)$. Note that σ is an automorphism. Clearly, R is not reduced and hence R is not σ-rigid. Moreover, R is not σ-skew power-serieswise Armendariz: For, $((2, 0) + (2, 1)x)^2 = 0 \in R[[x; \sigma]]$, but $(2, 0)(2, 1) \neq 0$.

Now, we show that R has strongly σ-skew IFP. Let $AB = 0$ for $A = (a, b), B = (c, d) \in R$. Then $ac = 0$ and $ad + bc = 0$. From $ac = 0$, we have three cases of (i) $a = 0$, (ii) $c = 0$, or (iii) $a = 2$ and $c = 2$. For any $(h, k) \in R$, $(a, b)(h, k)\sigma((c, d)) = (ahc, ah(-d) + akc + bhc)$. If $a = 0$ then $bc = 0$ and so $bhc = 0$, entailing that $AR\sigma(B) = 0$. If $c = 0$ then $ad = 0$ and so $ah(-d) = 0$, proving that $AR\sigma(B) = 0$. Finally, assume that $a = 2$ and $c = 2$. Then we also have $AR\sigma(B) = 0$ from $2b = 2d$.

Consequently, R has σ-skew IFP. By the similar computation to the above, we can easily show that R has IFP. Therefore R has strongly σ-skew IFP.

The following example shows that there exists a σ-skew power-serieswise Armendariz ring that does not have IFP (hence does not have strongly σ-skew IFP).

Example 3.2. Let S be a reduced ring and consider the subring

$$R = \left\{ \begin{pmatrix} a & b & c & d \\ 0 & a & e & f \\ 0 & 0 & a & g \\ 0 & 0 & 0 & a \end{pmatrix} \mid a, b, c, d, e, f, g \in S \right\}$$

of the 4 by 4 full matrix ring over S. Define an endomorphism σ of R by

$$\sigma\left(\begin{pmatrix} a & b & c & d \\ 0 & a & e & f \\ 0 & 0 & a & g \\ 0 & 0 & 0 & a \end{pmatrix}\right) = \begin{pmatrix} a & b & c & 0 \\ 0 & a & e & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}.$$

Then $\sigma(A) = \sigma^i(A)$ for all $A \in R$ and $i \geq 2$. Suppose that $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} A_i x^i, q(x) = \sum_{j=0}^{\infty} B_j x^j \in R[[x;\sigma]]$, where

$$A_i = \begin{pmatrix} a_i & b_i & c_i & d_i \\ 0 & a_i & e_i & f_i \\ 0 & 0 & a_i & g_i \\ 0 & 0 & 0 & a_i \end{pmatrix} \text{ and } B_j = \begin{pmatrix} k_j & r_j & s_j & t_j \\ 0 & k_j & u_j & v_j \\ 0 & 0 & k_j & w_j \\ 0 & 0 & 0 & k_j \end{pmatrix}$$

for each i and j. Set $p'(x) = \sum_{i=0}^{\infty} A'_i x^i, q'(x) = \sum_{j=0}^{\infty} B'_j x^j \in R[[x;\sigma]]$ where $A'_i = \sigma(A_i)$ and $B'_j = \sigma(B_j)$ for each i and j. Note that for

$$A = \begin{pmatrix} a & b & c & d \\ 0 & a & e & f \\ 0 & 0 & a & g \\ 0 & 0 & 0 & a \end{pmatrix} \in R,$$

letting $A = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix}$ with $A_{11} = \begin{pmatrix} a & b & c \\ 0 & a & e \\ 0 & 0 & a \end{pmatrix}$, $A_{12} = \begin{pmatrix} d \\ f \\ g \end{pmatrix}$, and $A_{22} = \begin{pmatrix} a \end{pmatrix}$, we have

$$\sigma(R) \cong \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \mid a, b, c, d \in S \right\}$$

via $\sigma(A) \mapsto A_{11}$. Hence $p'(x)q'(x) = 0$ since we get

$$\begin{aligned} 0 &= \sigma(A_0)\sigma(B_n) + \sigma(A_1)\sigma(B_{n-1}) + \sigma(A_2)\sigma(B_{n-2}) \\ &\quad + \cdots + \sigma(A_{n-2})\sigma(B_2) + \sigma(A_{n-1})\sigma(B_1) + \sigma(A_n)\sigma(B_0) \end{aligned} \tag{1}$$

from the equalities

$$\begin{aligned} 0 &= A_0 B_n + A_1\sigma(B_{n-1}) + A_2\sigma^2(B_{n-2}) \\ &\quad + \cdots + A_{n-2}\sigma^{n-2}(B_2) + A_{n-1}\sigma^{n-1}(B_1) + A_n\sigma^n(B_0) \\ &= A_0 B_n + A_1\sigma(B_{n-1}) + A_2\sigma(B_{n-2}) \\ &\quad + \cdots + A_{n-2}\sigma(B_2) + A_{n-1}\sigma(B_1) + A_n\sigma(B_0) \end{aligned} \tag{2}$$

where $n \geq 0$. Here we also have $p'(x)q'(x) = 0$ as a product of $p'(x)$ and $q'(x)$ in $\sigma(R)[[x]]$. But since $\sigma(R)$ is power-serieswise Armendariz by applying [15, Corollary 3.6(2)], we get

(3) $$A'_i B'_j = 0 \text{ for all } i, j.$$

Further, by the proof of [17, Proposition 1.2], we have that

$$a_i k_j = b_i k_j = c_i k_j = e_i k_j = 0$$

for all i, j from the equality (3). We will use this freely.

From the equality (2), we also have

(4) $$a_0 t_n + b_0 v_n + c_0 w_n + d_0 k_n + d_1 k_{n-1} + \cdots + d_n k_0 = 0$$

in the $(1, 4)$-entry of the coefficient of degree n of $p(x)q(x) = 0$, where $n \geq 0$. Multiplying the equality (4) by k_l on the right side, we obtain

(5) $$d_0 k_n k_l + d_1 k_{n-1} k_l + \cdots + d_n k_0 k_l = 0$$

where l is any nonnegative integer. Let

$$\alpha(x) = \sum_{i=0}^{\infty} d_i x^i, \beta(x) = \sum_{j=0}^{\infty} k_j k_l x^j$$

in $S[[x]]$. Then $\alpha(x)\beta(x) = 0$, so we have $d_i k_j k_l = 0$ for all i, j from the equality (5) since S is reduced (hence power-serieswise Armendariz). Especially we get $d_i k_j k_j = 0$ if we let $l = j$, so

(6) $$d_i k_j = 0 \text{ for all } i, j.$$

Through similar computations, we also obtain

(7) $$f_i k_j = 0 \text{ and } g_i k_j = 0 \text{ for all } i, j$$

in the $(2, 4)$, $(3, 4)$-entries of the coefficients of $p(x)q(x) = 0$.

Now from the results (3), (6), and (7), we can have

$$A_i \sigma^i(B_j) = 0 \text{ for all } i \geq 1 \text{ and } j \geq 0.$$

This yields that $A_0 B_j = 0$ for all $j \geq 0$. Consequently $A_i \sigma^i(B_j) = 0$ for all i, j, and thus R is σ-skew power-serieswise Armendariz. However R does not have IFP by [17, Example 1.3].

Based on the preceding example, we have the following.

Proposition 3.3. *Let R be a semiprime ring and σ a monomorphism of R. If R has σ-skew IFP, then R has IFP (hence strongly σ-skew IFP).*

Proof. Let R have σ-skew IFP. Suppose $ab = 0$ for $a, b \in R$. Then $aR\sigma(b) = 0$. But since R is semiprime, we get $\sigma(b)Ra = 0$. Thus $\sigma(b)R\sigma(a) = 0$, so the semiprimeness of R implies $\sigma(a)R\sigma(b) = 0$. Since σ is a monomorphism, we get $aRb = 0$ from $\sigma(aRb) \subseteq \sigma(a)R\sigma(b)$. ∎

As a corollary, we generalize the result of [2, Theorem 2.4].

Corollary 3.4. (1) *Let a ring R have σ-skew IFP for a monomorphism σ of R. Then R is semiprime if and only if R is reduced.*

(2) *R is a σ-rigid ring if and only if R is a semiprime ring that has σ-skew IFP and σ is a monomorphism.*

Proof. Note that a semiprime ring has IFP if and only if reduced. ∎

Proposition 3.5. *Let R be a ring with an endomorphism σ. Then R is a σ-skew power-serieswise Armendariz ring if and only if for every $p(x) = \sum_{i=0}^{\infty} a_i x^i$ and $q(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$, $p(x)q(x) = 0$ implies $a_0 b_j = 0$ for all j.*

Proof. We here adapt the proof of [21, Theorem 2.2], extending skew polynomials to skew power series. ∎

Corollary 3.6. (1) *Every skew power-serieswise σ-Armendariz ring is σ-skew power-serieswise Armendariz.*

(2) *A ring R is power-serieswise Armendariz if and only if for every $f(x) = \sum_{i=0}^{\infty} a_i x^i$ and $g(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x]]$, $f(x)g(x) = 0$ implies $a_0 b_j = 0$ for any j.*

Every domain with a monomorphism σ is σ-rigid by a simple computation, and moreover the condition "σ is a monomorphism" is not superfluous by [2, Example 2.5(2)].

Proposition 3.7. *Let R be a domain with an endomorphism σ. Then R is σ-skew power-serieswise Armendariz.*

Proof. Let $p(x)q(x) = 0$ where $p(x) = \sum_{i=0}^{\infty} a_i x^i, q(x) = \sum_{i=1}^{\infty} b_j x^j \in R[[x;\sigma]]$. From $a_0 b_0 = 0$, we have $a_0 = 0$ or $b_0 = 0$. If $a_0 = 0$, then $a_0 b_j = 0$ for all j. Hence R is σ-skew power-serieswise Armendariz by Proposition 3.5. Next suppose $a_0 \neq 0$. Then $b_0 = 0$. Since $0 = a_0 b_1 + a_1\sigma(b_0) = a_0 b_1$, we have $b_1 = 0$. From $0 = a_0 b_2 + a_1\sigma(b_1) + a_2\sigma^2(b_0) = a_0 b_2$, we obtain $b_2 = 0$. Inductively assume $b_0 = b_1 = \cdots = b_s = 0$. Then from $0 = a_0 b_{s+1} + a_1\sigma(b_s) + \cdots + a_s\sigma^s(b_1) + a_{s+1}\sigma^{s+1}(b_0) = a_0 b_{s+1}$, we get $b_{s+1} = 0$. This yields $q(x) = 0$. Consequently we get that $a_0 b_j = 0$ for all j in any case. Therefore R is σ-skew power-serieswise Armendariz by Proposition 3.5. ∎

The following example shows that the converse of Corollary 3.6(1) does not hold and the conclusion of Proposition 3.7 cannot be replaced by "R is skew power-serieswise σ-Armendariz".

Example 3.8. Let $R = \mathbb{Z}_2[x]$ with an endomorphism $\sigma : R \to R$ defined by $\sigma(f(x)) = f(0)$. Then R is not σ-Armendariz by [12, Example 1.9] and so not skew power-serieswise σ-Armendariz. Note that R is σ-skew power-serieswise Armendariz by Proposition 3.7.

A ring R is called *reversible* [5] if $ab = 0$ implies $ba = 0$ for $a, b \in R$. It is obvious that commutative rings and reduced rings are reversible and reversible rings have IFP, but not conversely in general. From an endomorphism σ of a ring R, we can induce an endomorphism $\bar{\sigma}$ of $R[[x;\sigma]]$ by defining $\bar{\sigma}(\sum_{i=0}^{\infty} a_i x^i) = \sum_{i=0}^{\infty} \sigma(a_i)x^i$.

Theorem 3.9. *Suppose that R is a σ-skew power-serieswise Armendariz ring. Then we have the following results.*

(1) *R has IFP if and only if $R[[x;\sigma]]$ has IFP if and only if $R[[x;\sigma]]$ has strongly $\bar{\sigma}$-skew IFP.*

(2) *Let σ be a monomorphism.* (i) *If R is reversible, then R is skew power-serieswise σ-Armendariz; and* (ii) *R is reversible if and only if $R[[x;\sigma]]$ is reversible.*

Proof. Let R be σ-skew power-serieswise Armendariz. Then R has σ-skew IFP by [11, Lemma 3.1(2)] and $\sigma(1) = 1$.

(1) It is enough to show that $R[[x;\sigma]]$ has strongly $\bar{\sigma}$-skew IFP when R has IFP, since the class of rings that have strongly σ-skew IFP is closed under subrings. Suppose that R has IFP. Then R has strongly σ-skew IFP by Lemma 2.5. Let $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} a_i x^i$, $q(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$. Then $a_i\sigma^i(b_j) = 0$ for all i, j, since R is σ-skew power-serieswise Armendariz. Hence $a_i R\sigma^{t+i}(b_j) = 0$ and so $a_i(Rx^t)\sigma^i(b_j) = 0$ for all i, j and $t \geq 0$ because R has strongly σ-skew IFP. This implies $p(x)(c_t x^t)q(x) = 0$ for any $c_t x^t \in R[[x;\sigma]]$ and all $t \geq 0$. Therefore $p(x)R[[x;\sigma]]q(x) = 0$, proving that $R[[x;\sigma]]$ has IFP. Since $R[[x;\sigma]]$ has IFP, $a_i\sigma^i(\sigma(b_j)) = 0$ and so $a_i R\sigma^{t+i}(\sigma(b_j)) = 0$ and equivalently, $a_i(Rx^t)\sigma^i(\sigma(b_j)) = 0$ for all i, j and $t \geq 0$, and hence $p(x)R[[x;\sigma]]\bar{\sigma}(q(x)) = 0$. Consequently, $p(x)R[[x;\sigma]]\bar{\sigma}^s(q(x)) = 0$ for any $s \geq 0$ and therefore $R[[x;\sigma]]$ has strongly $\bar{\sigma}$-skew IFP.

(2)-(i) Let R be reversible and $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} a_i x^i$, $q(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$. Then $a_i\sigma^i(b_j) = 0$ and $\sigma^i(b_j)a_i = 0$. Since R has σ-skew IFP, $\sigma^i(b_j)\sigma^i(a_i) = 0$ for all i, j by hypothesis. Thus $\sigma^i(b_j a_i) = 0$ and so $b_j a_i = 0$ for all i, j, since σ is a monomorphism. Hence $a_i b_j = 0$ for all i, j and therefore R is skew power-serieswise σ-Armendariz.

(ii) It suffices to show the necessity. Suppose that R is reversible. Let $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} a_i x^i$, $q(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$. Then $a_i b_j = 0$ for all i, j by (i) and so $b_j a_i = 0$, yielding that $b_j R\sigma^j(a_i) = 0$ since R has strongly σ-skew IFP. Then $b_j\sigma^j(a_i) = 0$ for all i, j and so $q(x)p(x) = 0$, proving that $R[[x;\sigma]]$ is reversible. ■

The hypothesis "R is a σ-skew power-serieswise Armendariz ring" in Theorem 3.9(1) cannot be dropped: Indeed, the ring R which has strongly σ-skew IFP of Example 2.8 is not σ-skew power-serieswise Armendariz and $R[[x;\sigma]]$ does not have $\bar{\sigma}$-skew IFP by the same argument as in Example 2.8.

Proposition 3.10. *If R is a skew power-serieswise σ-Armendariz ring, then R has strongly σ-skew IFP and $R[[x;\sigma]]$ has strongly $\bar{\sigma}$-skew IFP.*

Proof. We first show that R has strongly σ-skew IFP, applying the proof of [11, Lemma 3.1(2)]. Let R be a skew power-serieswise σ-Armendariz ring. Let $ab = 0$ for $a, b \in R$. Then for any $l \geq 1$ and $r \in R$, we have

$$\begin{aligned}0 = ab &= a(1 - rx^l)(1 + rx^l + r\sigma^l(r)x^{2l} + r\sigma^l(r)\sigma^{2l}(r)x^{3l} + \cdots)b \\ &= (a - arx^l)(b + r\sigma^l(b)x^l + r\sigma^l(r)\sigma^{2l}(b)x^{2l} + r\sigma^l(r)\sigma^{2l}(r)\sigma^{3l}(b)x^{3l} + \cdots),\end{aligned}$$

so we get $arb = 0$ and $ar\sigma^l(b) = 0$. Thus $aR\sigma^n(b) = 0$ for all $n \geq 0$, and so R has strongly σ-skew IFP. Therefore $R[[x;\sigma]]$ has strongly $\bar{\sigma}$-skew IFP by Theorem 3.9(1). ■

Note that if σ is an endomorphism of a ring R, then σ can be extended to the endomorphism $\bar{\sigma}$ of $n \times n$ full matrix ring $Mat_n(R)$ over R defined by $\bar{\sigma}((a_{ij})) = (\sigma(a_{ij}))$. It is well-known that for $n \geq 2$, $Mat_n(R)$ and the upper triangular matrix ring $U_n(R)$ over any ring R are not Abelian and so do not have IFP, and hence they do not have strongly $\bar{\sigma}$-skew IFP.

Let R be a reduced ring with an endomorphism σ and

$$D(R) = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \mid a, b, c, d \in R \right\}.$$

Then the ring R has (strongly) σ-skew IFP if and only if $D(R)$ has (strongly) $\bar{\sigma}$-skew IFP if and only if $T(R, R)$ has (strongly) $\bar{\sigma}$-skew IFP by combining [17, Proposition 1.2 and Proposition 1.6] and [2, Proposition 2.10 and Proposition 2.13].

Proposition 3.11. *Let R be a ring with a monomorphism σ. The following conditions are equivalent:*

(1) *R is a σ-rigid ring.*

(2) *$D(R)$ is a skew power-serieswise $\bar{\sigma}$-Armendariz ring.*

(3) *$D(R)$ is a $\bar{\sigma}$-skew power-serieswise Armendariz ring.*

(4) *$T(R, R)$ is a skew power-serieswise $\bar{\sigma}$-Armendariz ring.*

(5) *$T(R, R)$ is a $\bar{\sigma}$-skew power-serieswise Armendariz ring.*

Proof. (1)⇒(2): Suppose that R is a σ-rigid ring. Then $R[[x;\sigma]]$ is reduced by [8, Corollary 18]. For

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & a_1 & d_1 \\ 0 & 0 & a_1 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 & c_2 \\ 0 & a_2 & d_2 \\ 0 & 0 & a_2 \end{pmatrix} \in D(R),$$

we denote their addition and multiplication by

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \quad \text{and}$$

$$(a_1, b_1, c_1, d_1)(a_2, b_2, c_2, d_2) = (a_1a_2, a_1b_2 + b_1a_2, a_1c_2 + b_1d_2 + c_1a_2, a_1d_2 + d_1a_2),$$

respectively. So every $p(x) \in D(R)[[x;\bar{\sigma}]]$ can be expressed by the form of (p_0, p_1, p_2, p_3) for some p_i's in $R[[x;\sigma]]$. Let $p(x) = (p_0, p_1, p_2, p_3)$ and $q(x) = (q_0, q_1, q_2, q_3) \in D(R)[[x;\bar{\sigma}]]$ with $p(x)q(x) = 0$, where $p_0 = \sum_{i=0}^{\infty} a_i x^i, p_1 = \sum_{i=0}^{\infty} b_i x^i$, $p_2 = \sum_{i=0}^{\infty} c_i x^i$, $p_3 = \sum_{i=0}^{\infty} d_i x^i$, $q_0 = \sum_{j=0}^{\infty} a'_j x^j, q_1 = \sum_{j=0}^{\infty} b'_j x^j$, $q_2 = \sum_{j=0}^{\infty} c'_j x^j$ and $q_3 = \sum_{j=0}^{\infty} d'_j x^j$. By the same argument as in the proof of [9, Proposition 17], we can show that $D(R)$ is skew power-serieswise $\bar{\sigma}$-Armendariz.

(2)⇒(3) and (4)⇒(5) follow from Corollary 3.6(1).

(2)⇒(4) and (3)⇒(5) are obvious, since the class of skew power-serieswise σ-Armendariz (resp., σ-skew power-serieswise Armendariz) rings is clearly closed under subrings.

(5)⇒(1): Suppose that $T(R,R)$ is a $\bar{\sigma}$-skew power-serieswise Armendariz ring and $\bar{\sigma}$ is a monomorphism. Let $a\sigma(a) = 0$ for $a \in R$. We denote elements of $T(R,R)$ by (a,b). Note that $\sigma(1) = 1$. Consider the polynomials $p(x) = (0,1) + (-a,0)x$ and $q(x) = (0,1) + (a,0)x \in T(R,R)[[x;\bar{\sigma}]]$. Then $p(x)q(x) = 0$, and so $(0,1)(a,0) = 0$. Thus $a = 0$, entailing that R is σ-rigid. ■

Recall that there are many examples of non-reduced (and hence non-σ-rigid) skew power-serieswise σ-Armendariz rings as in [22, Section 4]. These examples and $D(R)$ in Proposition 3.11, which are skew power-serieswise $\bar{\sigma}$-(or σ-)Armendariz, are neither quasi-Baer nor p.p.-rings. But there also exist skew power-serieswise σ-Armendariz rings which are Baer or quasi-Baer. For example, letting K be a finite direct product of fields and σ be a monomorphism of K such that K has σ-skew IFP, $R[[x;\sigma]]$ ($R[x;\sigma]$) is Baer by Theorem 2.2 and the fact that left (or right) self-injective von Neumann regular rings are Baer [19, Proposition 4.1].

Recall that for an ideal I of R, if $\sigma(I) \subseteq I$ then $\bar{\sigma} : R/I \to R/I$ defined by $\bar{\sigma}(a+I) = \sigma(a)+I$ is an endomorphism of a factor ring R/I. The homomorphic image of a skew power-serieswise σ-Armendariz ring need not be $\bar{\sigma}$-skew power-serieswise Armendariz by the following example.

Example 3.12. We adapt the construction of [9, Example 7] to our argument. Let R be the trivial extension of $\mathbb{Z}$ by $\mathbb{Z}_4$, i.e., $R = T(\mathbb{Z}, \mathbb{Z}_4)$, and σ an endomorphism defined by $\sigma((a, s)) = (a, -s)$. Note that t_l is odd if and only if $(-1)^w t_l$ is odd for all $t_l \in \mathbb{Z}_4$ and $w \geq 1$. Let $h(x) \in \mathbb{Z}[[x]]$ and $k(x) \in \mathbb{Z}_4[[x]]$. In the product $h(x)k(x) = k(x)h(x)$, note that every coefficient of $h(x)$ can be identified by the remainder divided by 4. We will use these facts without mention. Note that every $f(x) = \sum_{i=0}^{\infty}(u_i, v_i)x^i \in R[[x;\sigma]]$ can be expressed by $f(x) = (f_0, f_1)$ with $f_0 = \sum_{i=0}^{\infty} u_i x^i \in \mathbb{Z}[[x]]$ and $f_1 = \sum_{i=0}^{\infty} v_i x^i \in \mathbb{Z}_4[[x]]$.

Let $p(x) = \sum_{i=0}^{\infty}(a_i, s_i)x^i = (p_0, p_1)$ and $q(x) = \sum_{j=0}^{\infty}(b_j, t_j)x^j = (q_0, q_1) \in R[[x;\sigma]]$. Then $p_0 = \sum_{i=0}^{\infty} a_i x^i$, $p_1 = \sum_{i=0}^{\infty} s_i x^i$, $q_0 = \sum_{j=0}^{\infty} b_j x^j$, and $q_1 = \sum_{j=0}^{\infty} t_j x^j$. Suppose $p(x)q(x) = 0$. Then $p_0 q_0 = 0$ in $\mathbb{Z}[[x]]$, and so $p_0 = 0$ or $q_0 = 0$ since $\mathbb{Z}[[x]]$ is a domain.

If $p_0 = 0$, then $p_1 q_0 = 0$ from $p(x)q(x) = 0$. Then $s_i b_j = 0$ for all i, j by [15, Proposition 3.2].

If $q_0 = 0$, then $p_0 q_1 = 0$ from $p(x)q(x) = 0$. We here can let $a_0 \neq 0$ and $t_0 \neq 0$ without loss of generality. We claim that both a_i and t_j are even for all i and j. Assume that i_0 and j_0 are minimal indices such that a_{i_0} and t_{j_0} are odd, respectively. Then we obtain

$$\begin{aligned} 0 = 2[&a_0 t_m + a_1((-1)t_{m-1}) + \cdots + a_{i_0-1}((-1)^{i_0-1}t_{j_0+1}) \\ &+ a_{i_0}((-1)^{i_0}t_{j_0}) \\ &+ a_{i_0+1}((-1)^{i_0+1}t_{j_0-1}) + \cdots + a_m((-1)^m t_0)] = 2a_{i_0}((-1)^{i_0}t_{j_0}), \end{aligned}$$

where $m = i_0 + j_0$. This entails that $a_{i_0}t_{j_0}$ is even, a contradiction.

Next assume that i_0 is the minimal index such that a_{i_0} is odd. Then every t_j must be even by the preceding computation; especially $t_0 = 2$. So we get

$$0 = a_0 t_{i_0} + a_1((-1)t_{i_0-1}) + \cdots + a_{i_0-1}((-1)^{i_0-1}t_1) + a_{i_0}((-1)^{i_0}t_0) = a_{i_0}((-1)^{i_0}t_0),$$

a contradiction since $a_{i_0}((-1)^{i_0}t_0) = 2$.

A contradiction also occurs similarly for the case that every a_i is even and some t_j is odd. Thus we must have that a_i and t_j are even for all i and j.

Therefore $(a_i, s_i)(b_j, t_j) = 0$ for all i, j in any case, and this implies that R is skew power-serieswise σ-Armendariz.

However, the factor ring $R/I \cong \{(a, b) \mid a, b \in \mathbb{Z}_4\}$, where $I = \{(a, 0) \mid a \in 4\mathbb{Z}\}$ is an ideal of R such that $\sigma(I) \subseteq I$, is not $\bar{\sigma}$-skew power-serieswise Armendariz as can be seen by the computation in [9, Example 7] that $((\bar{2}, \bar{0}) + (\bar{2}, \bar{1})x)^2 = 0$ but $(\bar{2}, 0)(\bar{2}, \bar{1}) \neq 0$ in $(R/I)[[x;\bar{\sigma}]]$.

The next example illuminates that there exists a ring R with an endomorphism σ such that for any ideal I of R with $\sigma(I) \subseteq I$, R/I is skew power-serieswise $\bar{\sigma}$-Armendariz and I is skew power-serieswise σ-Armendariz but R does not have σ-skew IFP.

Example 3.13. Consider the 2×2 upper triangular matrix ring $R = U_2(D)$ over a division ring D with an endomorphism σ defined by

$$\sigma\left(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\right) = \begin{pmatrix} a & -b \\ 0 & c \end{pmatrix}.$$

Then R does not have σ-skew IFP. For,

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = 0 \quad \text{but} \quad \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\sigma\left(\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\right) \neq 0.$$

Hence, R is not σ-skew power-serieswise Armendariz.

Next, note that the only nonzero proper ideals of R are

$$I_1 = \begin{pmatrix} D & D \\ 0 & 0 \end{pmatrix},\ I_2 = \begin{pmatrix} 0 & D \\ 0 & D \end{pmatrix} \text{ and } I_3 = \begin{pmatrix} 0 & D \\ 0 & 0 \end{pmatrix}$$

and $\sigma(I_i) \subseteq I_i$ for $i = 1, 2, 3$. Then R/I_1 and R/I_2 are isomorphic to D and so they are skew power-serieswise $\bar{\sigma}$-Armendariz by Theorem 3.9(2)-(i). The ring

$$R/I_3 = \left\{\begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix} + I_3 \mid a, c \in D\right\}$$

is a reduced ring (i.e., id_{R/I_3}-rigid), and hence each R/I_i (for $i = 1, 2, 3$) is skew power-serieswise $\bar{\sigma}$-Armendariz.

Notice that all I_i are not reduced as rings without identity. We show that each I_i (for $i = 1, 2, 3$) is skew power-serieswise σ-Armendariz. Clearly I_3 is power-serieswise σ-Armendariz.

Let $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} A_i x^i$ and $q(x) = \sum_{j=0}^{\infty} B_j x^j \in I_1[[x;\sigma]]$, where $A_i = \begin{pmatrix} a_i & b_i \\ 0 & 0 \end{pmatrix}$ and $B_j = \begin{pmatrix} c_j & d_j \\ 0 & 0 \end{pmatrix}$ for all i, j. Let $A_0 \neq 0$ and $B_0 \neq 0$ without loss of generality. From $A_0B_0 = 0$, we obtain $a_0c_0 = 0$ and $a_0d_0 = 0$. Since $B_0 \neq 0$, $a_0 = 0$ and so $A_0B_j = 0$ for all j. From $A_0B_1 + A_1\sigma(B_0) = 0$, we have $A_1\sigma(B_0) = 0$ and so $a_1 = 0$. Hence, $A_1\sigma(B_j) = 0$ and $A_1B_j = 0$ for all j. Inductively assume that $a_0 = a_1 = \ldots = a_s = 0$. Then $A_kB_j = 0 = A_k\sigma^k(B_j)$ for all $0 \leq k \leq s$ and j. Since $0 = A_0B_{s+1} + A_1\sigma(B_s) + \cdots + A_s\sigma^s(B_1) + A_{s+1}\sigma^{s+1}(B_0) = A_{s+1}\sigma^{s+1}(B_0)$, we get $a_{s+1} = 0$. Consequently, we get that $A_iB_j = 0$ for all i, j. Therefore I_1 is skew power-serieswise σ-Armendariz.

By the similar method to the above, we can show that I_2 is skew power-serieswise σ-Armendariz.

For a ring R, let $rAnn_R(2^R) = \{r_R(U) \mid U \subseteq R\}$ and recall the set of coefficients $C_{p(x)}$ of $p(x)$, for $p(x) \in R[[x;\sigma]]$.

Proposition 3.14. *Let σ be an endomorphism of a ring R. Then the following conditions are equivalent:*

(1) *R is skew power-serieswise σ-Armendariz.*

(2) *$\Phi : rAnn_R(2^R) \to rAnn_{R[[x;\sigma]]}(2^{R[[x;\sigma]]})$ is bijective with $\Phi(r_R(U)) = r_R(U)R[[x;\sigma]]$ for some $U \subseteq R$, and $a\sigma(b) = 0$ implies $ab = 0$ for $a, b \in R$.*

Proof. (1)⇒(2): We first claim that Φ is well-defined. Letting $U \subseteq R$ then $r_{R[[x;\sigma]]}(U) = r_R(U)R[[x;\sigma]]$, determining the map $\Phi : rAnn_R(2^R) \to rAnn_{R[[x;\sigma]]}(2^{R[[x;\sigma]]})$ with $\Phi(r_R(U)) = r_R(U)R[[x;\sigma]]$ for every $r_R(U) \in rAnn_R(2^R)$. We next show that Φ is injective. Put $\Phi(r_R(A)) = \Phi(r_R(B))$ for $A, B \subseteq R$. Then $r_R(A)R[[x;\sigma]] = r_R(B)R[[x;\sigma]]$ and so $r_{R[[x;\sigma]]}(A) = r_{R[[x;\sigma]]}(B)$ by the above. Thus

$$r_R(A) = r_{R[[x;\sigma]]}(A) \cap R = r_{R[[x;\sigma]]}(B) \cap R = r_R(B),$$

entailing that Φ is injective.

Let V be a subset of $R[[x;\sigma]]$ and C_V denote the set $\bigcup_{p(x)\in V} C_{p(x)}$. By the similar computation to the proof of [7, Proposition 3.1], we can show that Φ is surjective.

Now, let $p(x) \in V$. Then we have $r_{R[[x;\sigma]]}(p(x)) = r_{R[[x;\sigma]]}(C_{p(x)}) = r_R(C_{p(x)})R[[x;\sigma]]$, since R is skew power-serieswise σ-Armendariz. Hence, we get $r_R(p(x)) = r_R(C_{p(x)})$ and so $ax^k b = 0$ implies $ab = 0$ for $a, b \in R$ and $k \geq 1$.

(2)⇒(1): Let $p(x)q(x) = 0$ for $p(x) = \sum_{i=0}^{\infty} a_i x^i$ and $q(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\sigma]]$. Then $q(x) \in r_{R[[x;\sigma]]}(p(x)) = r_R(U)R[[x;\sigma]]$ for some $U \subseteq R$ by hypothesis. For any $c \in r_R(U)$, $p(x)c = 0$ and so $a_i\sigma^i(c) = 0$ for all i. By the condition, $a_i c = 0$ and this yields $a_i q(x) = 0$ since $b_j \in r_R(U)$ for all i and j. ■

Note. In Proposition 3.14, assume that $\Phi : rAnn_R(2^R) \to rAnn_{R[[x;\sigma]]}(2^{R[[x;\sigma]]})$ is bijective with $\Phi(r_R(U)) = r_R(U)R[[x;\sigma]]$ for some $U \subseteq R$. Then R has σ-skew IFP. To see this, let $ab = 0$ for $a, b \in R$. Then $a(1-rx)(1+rx+r\sigma(r)x^2+\cdots)b = 0$ for every $r \in R$. Let $p(x) = a(1-rx)$ and $q(x) = (1+rx+r\sigma(r)x^2+\cdots)b$. By assumption, there exists $U \subseteq R$ such that $r_{R[[x;\sigma]]}(p(x)) = r_R(U)[[x;\sigma]]$. This yields $q(x) \in r_R(U)[[x;\sigma]]$, so $b \in r_R(U)$ and $ar\sigma(b) = 0$. Thus $ab = 0$ implies $aR\sigma(b) = 0$.

Recall that a ring R is called *σ-compatible* [6] if $ab = 0 \Leftrightarrow a\sigma(b) = 0$ for $a, b \in R$. It can be easily checked that the endomorphism σ of a σ-compatible ring is clearly a monomorphism. Every skew power-serieswise σ-Armendariz ring is σ-compatible by Proposition 3.14. There exists a reduced ring but not σ-compatible as we see in $R = \mathbb{Z}_2 \oplus \mathbb{Z}_2$ with $\sigma((a,b)) = (b,a)$. In fact $(1,0)(0,1) = 0$ but $(1,0)\sigma((0,1)) = (1,0) \neq 0$.

In [10, Theorem 1], Hong et al. showed that for a ring R with an automorphism σ, if $r_{R[x;\sigma]}(A) \neq 0$ where A is a right ideal of $R[x;\sigma]$, then $r_R(A) \neq 0$. We have the following, replacing "σ is an automorphism" to "R is a σ-compatible ring".

Note. Let $S = R[x;\sigma]$ and suppose that R is a σ-compatible ring. If $r_S(f(x)S) \neq 0$ for $f(x) \in S$ then $r_R(f(x)S) \neq 0$.

Proof. We apply the method of Hirano in the proof of [7, Theorem 2.2]. Let $f(x) = \sum_{i=0}^{m} a_i x^i$ and $0 \neq g(x) = \sum_{j=0}^{n} b_j x^j \in r_S(f(x)S)$. If $f(x) = 0$ then we are done. Let $f(x)$ be a nonzero constant, a say. Then $aRg(x) \subset aSg(x) = 0$, entailing $aRb_j = 0$ for all j. Since R is σ-compatible, we get $aR\sigma^k(b_j) = 0$ and $aRx^k b_j = aR\sigma^k(b_j)x^k = 0$ for all $k \geq 1$. This yields $f(x)Sb_j = aSb_j = 0$ for all j.

Next let $deg(f(x)) \geq 1$, where $deg(f(x))$ is the degree of a polynomial $f(x) \in R[x;\sigma]$. Assume on the contrary that $r_R(f(x)S) = 0$, and let $g(x)$ be a polynomial of minimal degree in $r_S(f(x)S)$. From $f(x)Sg(x) = 0$, we get $f(x)Rg(x) = 0$ and so $a_m R\sigma^n(b_n) = 0$, equivalently $a_m R\sigma^k(b_n) = 0$ for all $k \geq 1$ by the σ-compatibility of R. This implies

$$a_m Sg(x) = a_m S(b_{n-1}x^{n-1} + \cdots + b_0)$$

and

$$0 = f(x)Sg(x) \supseteq f(x)S(a_m Sg(x)) = f(x)S(a_m S(b_{n-1}x^{n-1} + \cdots + b_0)).$$

So $a_m R(b_{n-1}x^{n-1} + \cdots + b_0) \subset r_S(f(x)S)$, and this forces $a_m R(b_{n-1}x^{n-1} + \cdots + b_0) = 0$ since $g(x)$ is of minimal degree in $r_S(f(x)S)$. We moreover obtain

$$a_m x^m Sg(x) \subset a_m Sg(x) = a_m S(b_{n-1}x^{n-1} + \cdots + b_0) = 0$$

by the σ-compatibility of R, entailing $(a_{m-1}x^{m-1} + \cdots + a_0)Sg(x) = 0$. Now we repeat the same computation on $(a_{m-1}x^{m-1} + \cdots + a_0)$ and $g(x)$. Then we obtain

$$\begin{aligned} &a_{m-1}x^{m-1}Sg(x) \\ &= a_{m-1}x^{m-1}S(b_{n-1}x^{n-1} + \cdots + b_0) \subset a_{m-1}S(b_{n-1}x^{n-1} + \cdots + b_0) = 0. \end{aligned}$$

Inductively we finally obtain that $a_i x^i Sg(x) = 0$ (hence $a_i x^i Rg(x) = 0$) for all i. This also yields that $a_i R\sigma^i(b_j) = 0$ (equivalently, $a_i R\sigma^k(b_j) = 0$ for all $k \geq 0$ by the σ-compatibility of R) for all i, j. This implies $f(x)Sb_j = 0$ for all j, proving our claim. ∎

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