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Erratum to: Two Optimal Inequalities for Anti-holomorphic Submanifolds and Their Applications

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Abstract. Theorem 4.1 of [1] is not correctly stated. In this erratum we make a correction on Theorem 4.1. As a consequence, we also make the corresponding correction on Theorem 5.2 of [1].

We follow the notation from the original article [1]. Theorem 4.1 of [1] is not correctly stated. The correct statement shall read as follows.

Theorem 4.1. *Let N be an anti-holomorphic submanifold of a complex space form $\widetilde{M}^{h+p}(4c)$ with $h = \operatorname{rank}_{\mathbf{C}} \mathcal{D} \geq 1$ and $p = \operatorname{rank} \mathcal{D}^{\perp} \geq 2$. Then we have*

$$\delta(\mathcal{D}) \leq \frac{(2h+p)^2}{2} H^2 + \frac{p}{2}(4h+p-1)c - \frac{3p^2}{2(p+2)} |H_{\mathcal{D}^{\perp}}|^2. \tag{4.3}$$

The equality sign of (4.3) holds identically if and only if the following three conditions are satisfied:

(a) *N is $\mathcal{D}$-minimal, i.e., $\overrightarrow{H}_{\mathcal{D}} = 0$,*

(b) *N is mixed totally geodesic, and*

(c) *there exists an orthonormal frame $\{e_{2h+1}, \ldots, e_n\}$ of $\mathcal{D}^{\perp}$ such that the second fundamental σ of N satisfies*

$$\begin{cases} \sigma^r_{rr} = 3\sigma^r_{ss} & \text{for } 2h+1 \leq r \neq s \leq 2h+p, \\ \sigma^r_{st} = 0 & \text{for distinct } r,s,t \in \{2h+1, \ldots, 2h+p\}. \end{cases} \tag{4.4}$$

This correction shall be made since the last formula (4.13) in the proof of Theorem 4.1 contains an error. The corrected (4.13) shall read as

$$\begin{aligned} &\frac{(2h+p)^2}{2} H^2 + \frac{p}{2}(4h+p-1)c - \delta(\mathcal{D}) \\ &\geq 2h^2 |\overrightarrow{H}_{\mathcal{D}}|^2 + \sum_{i=1}^{2h} \sum_{r=2h+1}^{2h+p} \|\sigma(e_i, e_r)\|^2 + \frac{3p^2}{2(p+2)} |H_{\mathcal{D}^{\perp}}|^2 \\ &\geq \frac{3p^2}{2(p+2)} |H_{\mathcal{D}^{\perp}}|^2. \end{aligned} \tag{4.13}$$

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Consequently, Theorem 5.2 in [1] shall be replaced by

Theorem 5.2. *Let N be an anti-holomorphic submanifold in a complex space form $\widetilde{M}^{1+p}(4c)$ with $h = \operatorname{rank}_{\mathbf{C}} \mathcal{D} = 1$ and $p = \operatorname{rank} \mathcal{D}^{\perp} \geq 2$. Then we have*

$$\delta(\mathcal{D}) \leq \frac{(p+2)^2}{2} H^2 + \frac{p(p+3)}{2} c - \frac{3p^2}{2(p+2)} |H_{\mathcal{D}^{\perp}}|^2. \tag{5.4}$$

The equality case of (5.4) holds identically if and only if $c = 0$ and either

(i) *N is a totally geodesic anti-holomorphic submanifold of $\mathbf{C}^{h+p}$ or,*

(ii) *up to dilations and rigid motions, N is given by an open portion of the following product immersion:*

$$\phi \colon \mathbf{C} \times S^p(1) \to \mathbf{C}^{1+p}; \quad (z, x) \mapsto (z, w(x)), \quad z \in \mathbf{C},\ x \in S^p(1),$$

where $w \colon S^p(1) \to \mathbf{C}^p$ is the Whitney p-sphere.

References

[1] F. R. Al-Solamy, B.-Y. Chen and S. Deshmukh, *Two optimal inequalities for anti-holomorphic submanifolds and their applications*, Taiwanese J. Math. **18** (2014), no. 1, 199–217.

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