Research Article

Some Inequalities of Simpson Type for h-Convex Functions via Fractional Integrals

Marian Matłoka

Department of Applied Mathematics, Poznań University of Economics, Aleja Niepodległości 10, 61-875 Poznań, Poland

Correspondence should be addressed to Marian Matłoka; marian.matloka@ue.poznan.pl

We establish some inequalities of Simpson type involving Riemann-Liouville fractional integrals for mappings whose first derivatives are h-convex.

1. Introduction

The following inequality is well known in the literature as Simpson's inequality.

Theorem 1. *Let* $f : [a, b] \rightarrow R$ *be four times continuously differentiable mapping on* (a, b) *and* $\|f^{(4)}\|_{\infty} = \sup_{x \in (a,b)} |f^{(4)}(x)| < \infty$.

Then, the following inequality holds:

$$\left| \frac{1}{3} \left[\frac{f(a) + f(b)}{2} + 2f \left(\frac{a+b}{2} \right) \right] - \frac{1}{b-a} \int_a^b f(x)\,dx \right| \tag{1}$$
$$\leq \frac{1}{2880} \left\| f^{(4)} \right\|_{\infty} (b-a)^4.$$

In [1], Dragomir et al. proved the following inequality.

Theorem 2. *Suppose* $f : [a, b] \rightarrow R$ *is a differentiable mapping whose derivative is continuous on* (a, b) *and* $f' \in L_1[a, b]$. *Then the following inequality holds:*

$$\left| \frac{1}{3} \left[\frac{f(a) + f(b)}{2} + 2f \left(\frac{a+b}{2} \right) \right] \right.$$
$$\left. - \frac{1}{b-a} \int_a^b f(x)\,dx \right| \leq \frac{b-a}{3} \left\| f' \right\|_1, \tag{2}$$

where $\|f'\|_1 = \int_a^b |f'(x)|\,dx$.

In [2], Sarikaya et al. obtained inequalities for differentiable convex mappings. The main inequality is as follows.

Theorem 3. *Let* $f : I \subset R \rightarrow R$ *be a differentiable mapping on* I° *such that* $f' \in L_1[a, b]$, *where* $a, b \in I^{\circ}$ *with* $a < b$. *If* $|f'|^q$ *is convex on* $[a, b]$, $q > 1$, *then the following inequality holds:*

$$\left| \frac{1}{6} \left[f(a) + 4f \left(\frac{a+b}{2} \right) + f(b) \right] - \frac{1}{b-a} \int_a^b f(x)\,dx \right|$$
$$\leq \frac{b-a}{12} \left(\frac{1 + 2^{p+1}}{3(p+1)} \right)^{1/p}$$
$$\cdot \left\{ \left(\frac{3 \left| f'(b) \right|^q + \left| f'(a) \right|^q}{4} \right)^{1/q} \right.$$
$$\left. + \left(\frac{\left| f'(b) \right|^q + 3 \left| f'(a) \right|^q}{4} \right)^{1/q} \right\}, \tag{3}$$

where $1/p + 1/q = 1$.

In [3], Sarikaya et al. obtained the following inequality for s-convex function.

Theorem 4. *Let $f : I \subset [0, \infty] \rightarrow R$ be a differentiable mapping on I° such that $f' \in L_1[a, b]$, where $a, b \in I^{\circ}$ with $a < b$. If $|f'|^q$ is s-convex on $[a, b]$, for some fixed $s \in [0, 1]$ and $q > 1$, then the following inequality holds:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] - \frac{1}{b-a} \int_a^b f(x)\, dx \right|$$

$$\leq \frac{(b-a)}{12} \left(\frac{1 + 2^{p+1}}{3(p+1)} \right)^{1/p}$$

$$\cdot \left\{ \left(\frac{|f'(b)|^q + |f'((a+b)/2)|^q}{s+1} \right)^{1/q} \right. \tag{4}$$

$$\left. + \left(\frac{|f'(a)|^q + |f'((a+b)/2)|^q}{s+1} \right)^{1/q} \right\},$$

where $1/p + 1/q = 1$.

For recent refinements, counterparts, generalizations, and inequalities of Simpson type, see [1–7].

In 2007, Varošanec in [8] introduced a large class of functions, the so-called h-convex functions. This class contains several well-known classes of functions such as nonnegative convex functions and s-convex functions. This class is defined in the following way: a function $f : I \rightarrow R$, $\emptyset \neq I \subset R$ being an interval, is called h-convex if

$$f(tx + (1-t)y) \leq h(t) f(x) + h(1-t) f(y) \tag{5}$$

holds for all $x, y \in I, t \in (0, 1)$, where $h : J \rightarrow R, h \not\equiv 0$, and J is an interval, $(0, 1) \subseteq J$.

The aim of this paper is to establish inequalities of Simpson type for h-convex mappings via fractional integrals which are defined in the following way: left-sided and right-sided Riemann-Liouville fractional integrals of the order $\alpha > 0$ are defined by

$$I_{a^+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)\, dt$$

$$(0 \leq a < x \leq b),$$

$$\tag{6}$$

$$I_{b^-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t)\, dt$$

$$(0 \leq a \leq x < b),$$

where $\Gamma(\alpha) = \int_0^{\infty} e^{-u} u^{\alpha-1}\, du$ is the gamma function. Here is $I_{a^+}^{\circ} f(x) = I_{b^-}^{\circ} f(x) = f(x)$.

2. Main Results

To prove our main results, we consider the following lemma.

Lemma 5. *Let $f : I \subset R \rightarrow R$ be an absolutely continuous mapping on I° such that $f' \in L_1[a, b]$, where $a, b \in I^{\circ}$ with $a < b$. Then the following inequality holds:*

$$\frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

$$- \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[I_{a^+}^{\alpha} f\left(\frac{a+b}{2}\right) + I_{b^-}^{\alpha} f\left(\frac{a+b}{2}\right) \right]$$

$$= \frac{b-a}{2} \left[\int_0^1 \left(\frac{t^{\alpha}}{2} - \frac{1}{3} \right) f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) dt \right. \tag{7}$$

$$\left. + \int_0^1 \left(\frac{1}{3} - \frac{t^{\alpha}}{2} \right) f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) dt \right].$$

Proof. By integration by parts and by the change of the variables, we have

$$\int_0^1 \left(\frac{t^{\alpha}}{2} - \frac{1}{3} \right) f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) dt$$

$$= \frac{2}{b-a} \left(\frac{t^{\alpha}}{2} - \frac{1}{3} \right) f\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) \bigg|_0^1$$

$$- \frac{\alpha}{b-a} \int_0^1 t^{\alpha-1} f\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) dt$$

$$= \frac{1}{3(b-a)} f(b) + \frac{2}{3(b-a)} f\left(\frac{a+b}{2}\right) \tag{8}$$

$$- \frac{2^{\alpha} \cdot \alpha}{(b-a)^{\alpha+1}} \int_{(a+b)/2}^b \left(u - \frac{a+b}{2} \right)^{\alpha-1} f(u)\, du$$

$$= \frac{1}{3(b-a)} f(b) + \frac{2}{3(b-a)} f\left(\frac{a+b}{2}\right)$$

$$- \frac{2^{\alpha}\Gamma(\alpha+1)}{(b-a)^{\alpha+1}} I_{b^-}^{\alpha} f\left(\frac{a+b}{2}\right).$$

Similarly, we have

$$\int_0^1 \left(\frac{1}{3} - \frac{t^{\alpha}}{2} \right) f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) dt$$

$$= \frac{2}{a-b} \left(\frac{1}{3} - \frac{t^{\alpha}}{2} \right) f\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) \bigg|_0^1$$

$$+ \frac{\alpha}{a-b} \int_0^1 t^{\alpha-1} f\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) dt$$

$$= \frac{1}{3(b-a)} f(a) + \frac{2}{3(b-a)} f\left(\frac{a+b}{2}\right) \tag{9}$$

$$+ \frac{2^{\alpha} \cdot \alpha}{(b-a)^{\alpha+1}} \int_{(a+b)/2}^a \left(\frac{a+b}{2} - u \right)^{\alpha-1} f(u)\, du$$

$$= \frac{1}{3(b-a)} f(a) + \frac{2}{3(b-a)} f\left(\frac{a+b}{2}\right)$$

$$- \frac{2^{\alpha}\Gamma(\alpha+1)}{(b-a)^{\alpha+1}} I_{a^+}^{\alpha} f\left(\frac{a+b}{2}\right).$$

From (8) and (9), we get (7). This completes the proof. $\square$

The following theorems give a new result of Simpson's inequality for h-convex functions via fractional integrals.

Theorem 6. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a, b \in I^\circ$ with $a < b$. If $|f'|$ is h-convex on $[a,b]$, then the following inequality holds:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right| \quad (10)$$
$$\leq \frac{b-a}{3} \left[|f'(a)| + |f'(b)| \right] \int_0^1 h(t)\,dt.$$

Proof. From Lemma 5 and since $|f'|$ is h-convex on $[a,b]$, we get

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|$$
$$\leq \frac{b-a}{2} \int_0^1 \left[\left| \frac{t^\alpha}{2} - \frac{1}{3} \right| \left| f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) \right| \right.$$
$$\left. + \left| \frac{1}{3} - \frac{t^\alpha}{2} \right| \left| f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) \right| \right] dt \leq \frac{b-a}{2}$$
$$\cdot \int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right| \left[h\left(\frac{1+t}{2}\right) |f'(b)| \right. \quad (11)$$
$$+ h\left(\frac{1-t}{2}\right) |f'(a)| + h\left(\frac{1+t}{2}\right) |f'(a)|$$
$$\left. + h\left(\frac{1-t}{2}\right) |f'(b)| \right] dt = \frac{b-a}{2} \left[|f'(a)| \right.$$
$$\left. + |f'(b)| \right] \int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right| \left[h\left(\frac{1+t}{2}\right) + h\left(\frac{1-t}{2}\right) \right] dt$$
$$\leq \frac{b-a}{3} \left[|f'(a)| + |f'(b)| \right] \int_0^1 h(t)\,dt,$$

where we used the fact that $|t^\alpha/2 - 1/3| \leq 1/3$ for all $x \in [0,1]$. The proof is completed. $\square$

Corollary 7. *If in Theorem 6 one takes $h(t) = t$ then inequality (10) reduces to the following inequality for the convex function:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right| \quad (12)$$
$$\leq \frac{b-a}{6} \left[|f'(a)| + |f'(b)| \right].$$

Corollary 8. *If in Theorem 6 one takes $h(t) = t^s$ then inequality (10) reduces to the following inequality for the s-convex function:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right| \quad (13)$$
$$\leq \frac{b-a}{3(s+1)} \left[|f'(a)| + |f'(b)| \right].$$

Corollary 9. *If in Theorem 6 one takes $h(t) = t$ and $\alpha = 1$ then from the proof of Theorem 6 it follows that the following inequality holds:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{1}{b-a} \int_a^b f(x)\,dx \right| \leq \frac{5(b-a)}{72} \left[|f'(a)| \right. \quad (14)$$
$$\left. + |f'(b)| \right].$$

Theorem 10. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a, b \in I^\circ$ with $a < b$. If $|f'|^q$ is h-convex on $[a,b]$ and $q > 1$, then the following inequality holds:*

$$\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.$$
$$\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|$$
$$\leq \frac{b-a}{6} \left\{ \left[|f'(b)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right. \right. \quad (15)$$
$$\left. + |f'(a)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt \right]^{1/q}$$
$$+ \left[|f'(a)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right.$$
$$\left. \left. + |f'(b)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt \right]^{1/q} \right\}.$$

Proof. From Lemma 5 and the Hőlder inequality, we have

$$
\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.
$$

$$
\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|
$$

$$
\leq \frac{b-a}{2} \left\{ \left(\int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right|^p dt \right)^{1/p} \right.
$$

$$
\cdot \left(\int_0^1 \left| f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) \right|^q dt \right)^{1/q}
$$

$$
+ \left(\int_0^1 \left| \frac{1}{3} - \frac{t^\alpha}{2} \right|^p dt \right)^{1/p}
$$

$$
\left. \cdot \left(\int_0^1 \left| f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) \right|^q dt \right)^{1/q} \right\},
$$

(16)

where $1/p + 1/q = 1$.

Because $|f'|^q$ is h-convex, we have

$$
\int_0^1 \left| f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) \right|^q dt
$$

$$
\leq |f'(b)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt
$$

$$
+ |f'(a)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt,
$$

$$
\int_0^1 \left| f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) \right|^q dt
$$

$$
\leq |f'(a)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt
$$

$$
+ |f'(b)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt.
$$

(17)

Using the fact that $|t^\alpha/2 - 1/3| = |1/3 - t^\alpha/2| \leq 1/3$ for all $t \in [0,1]$ and using the last two inequalities in (16) we obtain (15).

This completes the proof of the theorem. $\qquad\square$

Corollary 11. *If in Theorem 10 one takes $h(t) = t$ then inequality (15) reduces to the following inequality for the convex function:*

$$
\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.
$$

$$
\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|
$$

$$
\leq \frac{b-a}{2} \left[\left(\frac{3}{4} |f'(b)|^q + \frac{1}{4} |f'(a)|^q \right)^{1/q} \right.
$$

$$
\left. + \left(\frac{1}{4} |f'(b)|^q + \frac{3}{4} |f'(a)|^q \right)^{1/q} \right].
$$

(18)

Corollary 12. *If in Theorem 10 one takes $h(t) = t^s$ then inequality (15) reduces to the following inequality for the s-convex function:*

$$
\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.
$$

$$
\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|
$$

$$
\leq \frac{b-a}{2} \left(\frac{2}{s+1} \right)^{1/q}
$$

$$
\cdot \left\{ \left(|f'(b)|^q \left(1 - \frac{1}{2^{s+1}}\right) + |f'(a)|^q \frac{1}{2^{s+1}} \right)^{1/q} \right.
$$

$$
\left. + \left(|f'(b)|^q \frac{1}{2^{s+1}} + |f'(a)|^q \left(1 - \frac{1}{2^{s+1}}\right) \right)^{1/q} \right\}.
$$

(19)

Theorem 13. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a, b \in I^\circ$ with $a < b$. If $|f'|^q$ is h-convex on $[a,b]$ and $q \geq 1$, then the following inequality holds:*

$$
\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.
$$

$$
\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|
$$

$$
\leq \frac{b-a}{2} \left\{ \left(|f'(b)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right. \right.
$$

$$
\left. + |f'(a)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt \right)^{1/q}
$$

$$
+ \left(|f'(a)|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right.
$$

$$
\left. \left. + |f'(b)|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt \right)^{1/q} \right\}.
$$

(20)

Proof. From Lemma 5 and the power mean inequality, we have that the following inequality holds:

$$
\left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right.
$$

$$
\left. - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha} \left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right) + I_{b^-}^\alpha f\left(\frac{a+b}{2}\right) \right] \right|
$$

$$
\leq \frac{b-a}{2} \left(\int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right| dt \right)^{1-1/q}
$$

$$
\cdot \left[\left(\int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right| \left| f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right) \right|^q dt \right)^{1/q} \right.
$$

$$
+ \left(\int_0^1 \left| \frac{1}{3} - \frac{t^\alpha}{2} \right| dt
$$

$$
\left. \cdot \left| f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) \right|^q dt \right)^{1/q} \right].
$$

(21)

By the h-convexity of $|f'|^q$ and using the fact that $|t^\alpha/2 - 1/3| = |1/3 - t^\alpha/2| \le 1/3$ for all $t \in [0, 1]$, we have

$$\int_0^1 \left| \frac{t^\alpha}{2} - \frac{1}{3} \right| \left| f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a \right) \right|^q dt$$

$$\le \frac{1}{3} \left(\left| f'(b) \right|^q \int_0^1 h\left(\frac{1+t}{2} \right) dt \right.$$

$$\left. + \left| f'(a) \right|^q \int_0^1 h\left(\frac{1-t}{2} \right) dt \right),$$

$$\int_0^1 \left| \frac{1}{3} - \frac{t^\alpha}{2} \right| \left| f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b \right) \right|^q dt$$

$$\le \frac{1}{3} \left(\left| f'(a) \right|^q \int_0^1 h\left(\frac{1+t}{2} \right) dt \right.$$

$$\left. + \left| f'(b) \right|^q \int_0^1 h\left(\frac{1-t}{2} \right) dt \right).$$

(22)

Using the last two inequalities in (21) we obtain (20). This completes the proof. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

References

[1] S. S. Dragomir, R. P. Agarwal, and P. Cerone, "On Simpson's inequality and applications," *Journal of Inequalities and Applications*, vol. 5, pp. 533–579, 2000.

[2] M. Z. Sarikaya, E. Set, and E. Özdemir, "On new inequalities of Simpson's type for convex functions," *RGMIA Research Report Collection*, vol. 13, no. 2, article 2, 2010.

[3] M. Z. Sarikaya, E. Set, and M. E. Özdemir, "On new inequalities of Simpson's type for s-convex functions," *Computers & Mathematics with Applications*, vol. 60, no. 8, pp. 2191–2199, 2010.

[4] M. Alomari, M. Darus, and S. S. Dragomir, "New inequalities of Simpson's type for s-convex functions with applications," *RGMIA Research Report Collection*, vol. 12, no. 4, article 9, 2009.

[5] M. Alomari and S. Hussain, "Two inequalities of Simpson type for quasi-convex functions and applications," *Applied Mathematics E-Notes*, vol. 11, pp. 110–117, 2011.

[6] Z. Liu, "An inequality of Simpson type," *Proceedings of The Royal Society of London. Series A. Mathematical, Physical and Engineering Sciences*, vol. 461, no. 2059, pp. 2155–2158, 2005.

[7] E. Set, M. E. Özdemir, and M. Z. Sarikaya, "On new inequalities of Simpson's type for quasi-convex functions with applications," *Tamkang Journal of Mathematics*, vol. 43, no. 3, pp. 357–364, 2012.

[8] S. Varošanec, "On h-convexity," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 1, pp. 303–311, 2007.