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Research Article

Global Attractivity of an Integrodifferential Model of Mutualism

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Sufficient conditions are obtained for the global attractivity of the following integrodifferential model of mutualism: $dN_1(t)/dt = r_1N_1(t)[((K_1+\alpha_1\int_0^\infty J_2(s)N_2(t-s)ds)/(1+\int_0^\infty J_2(s)N_2(t-s)ds)) - N_1(t)]$, $dN_2(t)/dt = r_2N_2(t)[((K_2+\alpha_2\int_0^\infty J_1(s)N_1(t-s)ds)/(1+\int_0^\infty J_1(s)N_1(t-s)ds)) - N_2(t)]$, where r_i, K_i, and α_i, $i = 1, 2$, are all positive constants. Consider $\alpha_i > K_i$, $i = 1, 2$. Consider $J_i \in C([0,+\infty),[0,+\infty))$ and $\int_0^\infty J_i(s)ds = 1$, $i = 1, 2$. Our result shows that conditions which ensure the permanence of the system are enough to ensure the global stability of the system. The result not only improves but also complements some existing ones.

1. Introduction

Mutualism, one of the most important relationships in the theory of ecology, however, was pointed out by Murray as follows [1]: "this area has not been as widely sutdied as the others even though its importance is comparable to that of predator-prey and competition interactions."

Traditional two species Lotka-Volterra take the form

$$\frac{dN_1}{dt} = r_1N_1\left(1-\frac{N_1}{K_1}+b_{12}\frac{N_2}{K_1}\right),$$
$$\frac{dN_2}{dt} = r_2N_2\left(1-\frac{N_2}{K_2}+b_{21}\frac{N_1}{K_2}\right). \quad (1)$$

Murray [1] gave detail analysis of the phase trajectories for the above system. He also pointed out that the system has certain drawback; one is the sensitivity between unbounded growth and a finite positive steady state. Despite the drawback of the system, since it is the most simple model on the mutualism, scholars incorporated delays to the above system and proposed the following system:

$$\frac{dx_1}{dt} = x_1(t)\left[r_1 - a_{11}x_1(t-\tau_{11}) + a_{12}x_2(t-\tau_{12})\right],$$
$$\frac{dx_2}{dt} = x_2(t)\left[r_2 + a_{21}x_1(t-\tau_{21}) - a_{22}x_2(t-\tau_{22})\right]. \quad (2)$$

Chen et al. [2] had given two examples to show that under the assumption $a_{11}a_{22} > a_{12}a_{21}$, a condition which could ensure the global stability of the system without delay, the system still admits unbounded solution. He and Gopalsamy [3] and Mukherjee [4] tried to investigate the persistent and stability property of the nonautonomous case of above system; however, in their main results, in addition to condition $a_{11}a_{22} > a_{12}a_{21}$, they further assumed that the density of one of the species should be bounded from above, such an assumption is by no means easy to verify. To overcome this difficulty, Lu et al. [5, 6] and Nakata and Muroya [7] tried to give restriction on the coefficients of the system or restriction on the delay of the system, and some interesting results about the permanence of Lotka-Volterra type mutualism system with delay were obtained. Liu et al. [8] and Lu [9] also investigated the positive periodic solution of the Lotka-Volterra type mutualism model.

On the other hand, stimulated by the functional response of the predator-prey system, Wright [10] proposed the following two species mutualism model:

$$\frac{dN}{dt} = r_1N\left[K_1 - c_1N + \frac{b_1a_1M}{1+a_1T_{h1}M}\right],$$
$$\frac{dM}{dt} = r_2M\left[K_2 - c_2N + \frac{b_2a_2N}{1+a_2T_{h2}N}\right]. \quad (3)$$

Obviously, the model could be revised as follows:

$$\frac{dN(t)}{dt} = r_1 N \left[\frac{K_1 + \alpha_1 M}{1 + a_1 T_{h1} M} - c_1 N \right],$$
$$\frac{dM(t)}{dt} = r_2 M \left[\frac{K_2 + \alpha_2 N}{1 + a_2 T_{h2} N} - c_2 M \right], \tag{4}$$

where $\alpha_1 = K_1 + a_1 T_{h1}$, $\alpha_2 = K_2 + a_2 T_{h2}$, and one could easily see that $\alpha_i > K_i$, $i = 1, 2$.

It is well known that in a more realistic model the delay effect should be an average over past populations. This results in an equation with a distributed delay or an infinite delay. Based on the model (4), Gopalsamy [11] further proposed the following two species mutualism model:

$$\frac{dN_1(t)}{dt} = r_1 N_1 \left[\frac{K_1 + \alpha_1 \int_0^\infty K_2(s) N_2(t-s)\,ds}{1 + \int_0^\infty K_2(s) N_2(t-s)\,ds} - N_1(t) \right],$$
$$\frac{dN_2(t)}{dt} = r_2 N_2 \left[\frac{K_2 + \alpha_2 \int_0^\infty K_1(s) N_1(t-s)\,ds}{1 + \int_0^\infty K_1(s) N_1(t-s)\,ds} - N_2(t) \right]. \tag{5}$$

However, the author did not investigate the dynamic behaviors of the system.

Recently, Li and Xu [12] proposed and studied the following nonautonomous case of the system (5):

$$\begin{aligned} \frac{dN_1(t)}{dt} &= r_1(t) N_1(t) \times \left[\frac{K_1(t) + \alpha_1(t) \int_0^\infty J_2(s) N_2(t-s)\,ds}{1 + \int_0^\infty J_2(s) N_2(t-s)\,ds} - N_1(t) \right], \\ \frac{dN_2(t)}{dt} &= r_2(t) N_2(t) \times \left[\frac{K_2(t) + \alpha_2(t) \int_0^\infty J_1(s) N_1(t-s)\,ds}{1 + \int_0^\infty J_1(s) N_1(t-s)\,ds} - N_2(t) \right], \end{aligned} \tag{6}$$

where r_i, K_i, α_i, and σ_i, $i = 1, 2$, are continuous functions bounded above and below by positive constants. Consider $\alpha_i > K_i$, $i = 1, 2$. Consider $J_i \in C([0, +\infty), [0, +\infty))$ and $\int_0^\infty J_i(s)ds = 1$, $i = 1, 2$. Under the assumption that r_i, K_i, and α_i, $i = 1, 2$, are continuous periodic functions with common period ω. $\alpha_i > K_i$, $i = 1, 2$, $J_i \in C([0, +\infty), [0, +\infty))$ and $\int_0^\infty J_i(s)ds = 1$, $i = 1, 2$. By applying the coincidence degree theory, they showed that system (6) admits at least one positive ω-periodic solution. Chen and You [13] argued that the general nonautonomous case is more suitable. Concerned with the persistent property of the system (6), by applying an integral inequality (see Lemma 3 in the next section), they obtained the following result.

Theorem A. *The system* (6) *is always permanent. That is, there exist constants m_i, M_i, $i = 1, 2$, which are independent of the solution of the system* (6), *such that*

$$m_i \le \liminf_{t \to +\infty} N_i(t) \le \limsup_{t \to +\infty} N_i(t) \le M_i, \quad i = 1, 2. \tag{7}$$

Such a result is a roughly one, since it only tells us that the solution is finally bounded above and below by positive constants and there is no fine description of the stable or unstable property of the solution, for example, whether the delay of the system could induce the Hopf bifurcation to period solution or not? Does the system admit some kind of chaotic behaviors? Is it difficult to obtain sufficient conditions which ensure the global attractivity of the positive solution? Indeed, to the best of the authors' knowledge, to this day, still no scholars investigate the stability property of the system (6), which is one of the most important topics in the study of population dynamics. Noting that system (6) is nonautonomous one and for such kind of system, generally speaking, by constructing some suitable Lyapunov functional, one could always obtain some sufficient conditions which ensure the stability of the system; however, the condition is not easy to verify [14]. This motivated us to investigate the stability property of the system (5).

From the point of view of biology, in the sequel, we will consider (5) together with the initial conditions

$$N_i(s) = \phi_i(s), \quad s \in (-\infty, 0], \; i = 1, 2, \tag{8}$$

where $\phi_i \in BC^+$ and

$$BC^+ = \{\phi \in C((-\infty, 0], [0, +\infty)) : \phi(0) > 0, \; \phi \text{ be bounded}\}, \quad i = 1, 2. \tag{9}$$

From [15], system (5) has a unique positive solution $(N_1(t), N_2(t))$ satisfying the initial condition (8).

The aim of this paper is, by further developing the analysis technique of Chen and You [13] and de Oca and Vivas [16] and using the differential inequality theory, to obtain a set of sufficient conditions to ensure the global attractivity of the system (5). More precisely, we will prove the following result.

Theorem 1. *System* (5) *admits a unique positive equilibrium (N_1^*, N_2^*), which is globally attractive; that is, for any positive solution $(N_1(t), N_2(t))$ of system* (5) *with the initial condition* (8), *one has*

$$\lim_{t \to +\infty} N_i(t) = N_i^*, \quad i = 1, 2. \tag{10}$$

We will prove this theorem in the next section and then give a brief discussion in Section 3. For more works on the mutualism or cooperation system, one could refer to [12–15, 17–21] and the references cited therein.

2. Proof of the Main Result

Now let us state several lemmas which will be useful in the proving of main result.

Lemma 2. *System* (5) *admits a unique positive equilibrium* (N_1^*, N_2^*).

Proof. The positive equilibrium of the system (5) satisfies the following equation:

$$\begin{gathered}\frac{K_1+\alpha_1 N_2}{1+N_2}-N_1=0,\\ \frac{K_2+\alpha_2 N_1}{1+N_1}-N_2=0.\end{gathered} \tag{11}$$

System (11) admits a unique positive solution (N_1^*, N_2^*), where

$$\begin{gathered}N_1^*=\frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1},\\ N_2^*=\frac{-B_2+\sqrt{B_2^2-4B_1B_3}}{2B_1},\\ A_1=1+\alpha_2,\\ A_2=K_2-K_1-\alpha_2\alpha_1+1,\\ A_3=-\alpha_1K_2-K_1,\\ B_1=1+\alpha_1,\\ B_2=K_1-K_2-\alpha_1\alpha_2+1,\\ B_3=-K_2-\alpha_2K_1.\end{gathered} \tag{12}$$

This ends the proof of Lemma 2. □

Following Lemma 3 is Lemma 3 of de Oca and Vivas [16].

Lemma 3. *Let* $x : R \to R$ *be a bounded nonnegative continuous function, and let* $k : [0,+\infty) \to (0,+\infty)$ *be a continuous kernel such that* $\int_0^\infty k(s)ds = 1$. *Then*

$$\begin{aligned}\liminf_{t\to+\infty} x(t) &\le \liminf_{t\to+\infty}\int_{-\infty}^{t} k(t-s)\,x(s)\,ds\\ &\le \limsup_{t\to+\infty}\int_{-\infty}^{t} k(t-s)\,x(s)\,ds\\ &\le \limsup_{t\to+\infty} x(t).\end{aligned} \tag{13}$$

As a direct corollary of Lemma 2.2 of Chen [22], we have the following lemma.

Lemma 4. *If* $a > 0$, $b > 0$, *and* $\dot{x} \ge x(b-ax)$, *when* $t \ge 0$ *and* $x(0) > 0$, *one has*

$$\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}. \tag{14}$$

If $a > 0$, $b > 0$ *and* $\dot{x} \le x(b-ax)$, *when* $t \ge 0$ *and* $x(0) > 0$, *one has*

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}. \tag{15}$$

Now we are in the position of proving the main result of this paper.

Proof of Theorem 1. Let $(N_1(t), N_2(t))$ be any positive solution of the system (5) with initial condition (8). Similarly to the analysis of (11)–(17) in [13], from the first equation of the system (5) it follows that

$$\frac{dN_1(t)}{dt} \le N_1(t)\left[r_1(K_1+\alpha_1)-r_1N_1(t)\right]. \tag{16}$$

Thus, as a direct corollary of Lemma 4, according to (16), one has

$$\limsup_{t\to+\infty} N_1(t) \le K_1+\alpha_1, \tag{17}$$

and so, from Lemma 3 we have

$$\limsup_{t\to+\infty}\int_0^\infty J_1(s)\,N_1(t-s)\,ds \le K_1+\alpha_1. \tag{18}$$

Hence, for enough small $\varepsilon > 0$, it follows from (17) and (18) that there exists a $T_1' > 0$ such that

$$\begin{aligned}N_1(t) &< K_1+\alpha_1+\varepsilon \overset{\text{def}}{=} M_1^{(1)},\\ \int_0^\infty J_1(s)\,N_1(t-s)\,ds &= \int_{-\infty}^{t} J_1(t-s)\,N_1(s)\,ds\\ &\le K_1+\alpha_1+\varepsilon \overset{\text{def}}{=} M_1^{(1)} \quad \text{for } t > T_1'.\end{aligned} \tag{19}$$

Similarly, for above $\varepsilon > 0$, it follows from the second equation of the system (5) that there exists a $T_1 > T_1'$ such that

$$\begin{aligned}N_2(t) &< K_2+\alpha_2+\varepsilon \overset{\text{def}}{=} M_2^{(1)},\\ \int_0^\infty J_2(s)\,N_2(t-s)\,ds &< K_2+\alpha_2+\varepsilon \overset{\text{def}}{=} M_2^{(1)}\\ &\qquad \text{for } t > T_1.\end{aligned} \tag{20}$$

Noting that the function $g_1(x) = ((K_1+\alpha_1x)/(1+x))$ $(\alpha_1 > K_1,\ x \ge 0)$ is a strictly increasing function, hence, (20) together with the first equation of the system (5) implies

$$\frac{dN_1(t)}{dt} < N_1(t)\left[\frac{r_1\left(K_1+\alpha_1M_2^{(1)}\right)}{1+M_2^{(1)}}-r_1N_1(t)\right] \quad \text{for } t > T_1. \tag{21}$$

Therefore, by Lemma 4, we have

$$\limsup_{t\to+\infty} N_1(t) \le \frac{K_1+\alpha_1M_2^{(1)}}{1+M_2^{(1)}}. \tag{22}$$

Thus, from Lemma 3 we have

$$\limsup_{t\to+\infty}\int_0^\infty J_1(s)\,N_1(t-s)\,ds \le \frac{K_1+\alpha_1M_2^{(1)}}{1+M_2^{(1)}}. \tag{23}$$

That is, for $\varepsilon > 0$ defined by (19), there exists a $T_2' > T_1$ such that

$$N_1(t) < \frac{K_1 + \alpha_1 M_2^{(1)}}{1 + M_2^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)} > 0,$$
$$\int_0^\infty J_1(s) N_1(t-s)\, ds < \frac{K_1 + \alpha_1 M_2^{(1)}}{1 + M_2^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)} > 0 \quad \text{for } t > T_2'. \tag{24}$$

Similarly to the analysis of (22)–(24), from (19) and the second equation of the system (5), there exists a $T_2 > T_2'$ such that

$$N_2(t) < \frac{K_2 + \alpha_2 M_1^{(1)}}{1 + M_1^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)} > 0,$$
$$\int_0^\infty J_2(s) N_2(t-s)\, ds < \frac{K_2 + \alpha_2 M_1^{(1)}}{1 + M_1^{(1)}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_2^{(2)} > 0 \quad \text{for } t > T_2. \tag{25}$$

Since the function $g_1(x) = ((K_1 + \alpha_1 x)/(1 + x))$ $(\alpha_1 > K_1,\ x \ge 0)$ is a strictly increasing function, one could easily see that $g(x) \ge g(0) = K_1$, and so, from the first equation of the system (5), it follows that

$$\frac{dN_1(t)}{dt} \ge N_1(t)\left[r_1 K_1 - r_1 N_1(t)\right]. \tag{26}$$

Thus, as a direct corollary of Lemma 4, according to (25), one has

$$\liminf_{t \to +\infty} N_1(t) \ge K_1, \tag{27}$$

and so, from Lemma 3, we have

$$\liminf_{t \to +\infty} \int_0^\infty J_1(s) N_1(t-s)\, ds \ge K_1. \tag{28}$$

Hence, for enough small $\varepsilon > 0$ ($\varepsilon < (1/2)\min\{K_1, K_2\}$), it follows from (27) and (28) that there exists a $T_3' > 0$ such that

$$N_1(t) > K_1 - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)},$$
$$\int_0^\infty J_1(s) N_1(t-s)\, ds \ge K_1 - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)} \quad \text{for } t > T_3'. \tag{29}$$

Similarly, for above $\varepsilon > 0$, it follows from the second equation of the system (5) that there exists a $T_3 > T_3'$ such that

$$N_2(t) > K_2 - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)},$$
$$\int_0^\infty J_2(s) N_2(t-s)\, ds > K_2 - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)} \quad \text{for } t > T_3. \tag{30}$$

Noting that the function $g_1(x) = ((K_1 + \alpha_1 x)/(1 + x))$ $(\alpha_1 > K_1)$ is a strictly increasing function, hence, (30) together with the first equation of the system (5) implies

$$\frac{dN_1(t)}{dt} > N_1(t)\left[\frac{r_1\left(K_1 + \alpha_1 m_2^{(1)}\right)}{1 + m_2^{(1)}} - r_1 N_1(t)\right] \quad \text{for } t > T_3. \tag{31}$$

Therefore, by Lemma 4, we have

$$\liminf_{t \to +\infty} N_1(t) \ge \frac{K_1 + \alpha_1 m_2^{(1)}}{1 + m_2^{(1)}}. \tag{32}$$

Thus, from Lemma 3 we have

$$\liminf_{t \to +\infty} \int_0^\infty J_1(s) N_1(t-s)\, ds \ge \frac{K_1 + \alpha_1 m_2^{(1)}}{1 + m_2^{(1)}}. \tag{33}$$

That is, there exists a $T_4' > T_3$ such that

$$N_1(t) > \frac{K_1 + \alpha_1 m_2^{(1)}}{1 + m_2^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)} > 0,$$
$$\int_0^\infty J_1(s) N_1(t-s)\, ds > \frac{K_1 + \alpha_1 m_2^{(1)}}{1 + m_2^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)} > 0 \quad \text{for } t > T_4'. \tag{34}$$

Similarly to the analysis of (32)–(34), from (28) and the second equation of the system (5), there exists a $T_4 > T_4'$ such that

$$N_2(t) > \frac{K_2 + \alpha_2 m_1^{(1)}}{1 + m_1^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)} > 0,$$
$$\int_0^\infty J_2(s) N_2(t-s)\, ds > \frac{K_2 + \alpha_2 m_1^{(1)}}{1 + m_1^{(1)}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_2^{(2)} > 0 \quad \text{for } t > T_4. \tag{35}$$

One could easily see that

$$\begin{aligned}
M_1^{(2)} &= \frac{K_1 + \alpha_1 M_2^{(1)}}{1 + M_2^{(1)}} + \frac{\varepsilon}{2} < K_1 + \alpha_1 + \varepsilon = M_1^{(1)}; \\
M_2^{(2)} &= \frac{K_2 + \alpha_2 M_1^{(1)}}{1 + M_1^{(1)}} + \frac{\varepsilon}{2} < K_2 + \alpha_2 + \varepsilon = M_2^{(1)}; \\
m_1^{(2)} &= \frac{K_1 + \alpha_1 m_2^{(1)}}{1 + m_2^{(1)}} - \frac{\varepsilon}{2} > K_1 - \varepsilon = m_1^{(1)}; \\
m_2^{(2)} &= \frac{K_2 + \alpha_2 m_1^{(1)}}{1 + m_1^{(1)}} - \frac{\varepsilon}{2} > K_2 - \varepsilon = m_2^{(1)}.
\end{aligned} \tag{36}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}$, $m_i^{(n)}$, $i = 1, 2$, $n = 1, 2, \ldots$, such that for $n \geq 2$

$$\begin{aligned} M_1^{(n)} &= \frac{K_1 + \alpha_1 M_2^{(n-1)}}{1 + M_2^{(n-1)}} + \frac{\varepsilon}{n}; \\ M_2^{(n)} &= \frac{K_2 + \alpha_2 M_1^{(n-1)}}{1 + M_1^{(n-1)}} + \frac{\varepsilon}{n}; \\ m_1^{(n)} &= \frac{K_1 + \alpha_1 m_2^{(n-1)}}{1 + m_2^{(n-1)}} - \frac{\varepsilon}{n}; \\ m_2^{(n)} &= \frac{K_2 + \alpha_2 m_1^{(n-1)}}{1 + m_1^{(n-1)}} - \frac{\varepsilon}{n}. \end{aligned} \tag{37}$$

Obviously,

$$m_i^{(n)} < N_i(t) < M_i^{(n)}, \quad \text{for } t \geq T_{2n}, \ i = 1, 2. \tag{38}$$

We claim that sequences $M_i^{(n)}$, $i = 1, 2$ are nonincreasing and sequences $m_i^{(n)}$, $i = 1, 2$ are nondecreasing. To prove this claim, we will carry out by induction. Firstly, from (36) we have

$$M_i^{(2)} < M_i^{(1)}, \quad m_i^{(2)} > m_i^{(1)}, \quad i = 1, 2. \tag{39}$$

Let us assume now that our claim is true for n; that is,

$$M_i^{(n)} < M_i^{(n-1)}, \quad m_i^{(n)} > m_i^{(n-1)}, \quad i = 1, 2. \tag{40}$$

Again from the strict increasing of the function $g_i(x) = ((K_i + \alpha_i x)/(1 + x))$ $(\alpha_i > K_i, \ i = 1, 2)$, we immediately obtain

$$\begin{aligned} M_1^{(n+1)} &= \frac{K_1 + \alpha_1 M_2^{(n)}}{1 + M_2^{(n)}} + \frac{\varepsilon}{n+1} \\ &< \frac{K_1 + \alpha_1 M_2^{(n-1)}}{1 + M_2^{(n-1)}} + \frac{\varepsilon}{n} = M_1^{(n)}; \\ M_2^{(n+1)} &= \frac{K_2 + \alpha_2 M_1^{(n)}}{1 + M_1^{(n)}} + \frac{\varepsilon}{n+1} \\ &< \frac{K_2 + \alpha_2 M_1^{(n-1)}}{1 + M_1^{(n-1)}} + \frac{\varepsilon}{n} = M_2^{(n)}; \\ m_1^{(n+1)} &= \frac{K_1 + \alpha_1 m_2^{(n)}}{1 + m_2^{(n)}} - \frac{\varepsilon}{n+1} \\ &> \frac{K_1 + \alpha_1 m_2^{(n-1)}}{1 + m_2^{(n-1)}} - \frac{\varepsilon}{n} = m_1^{(n)}; \\ m_2^{(n+1)} &= \frac{K_2 + \alpha_2 m_1^{(n)}}{1 + m_1^{(n)}} - \frac{\varepsilon}{n+1} \\ &> \frac{K_2 + \alpha_2 m_1^{(n-1)}}{1 + m_1^{(n-1)}} - \frac{\varepsilon}{n} = m_2^{(n)}. \end{aligned} \tag{41}$$

Therefore,

$$\lim_{t \to +\infty} M_i^{(n)} = \overline{N}_i, \quad \lim_{t \to +\infty} m_i^{(n)} = \underline{N}_i, \quad i = 1, 2. \tag{42}$$

Letting $n \to +\infty$ in (37), we obtain

$$\begin{aligned} \overline{N}_1 &= \frac{K_1 + \alpha_1 \overline{N}_2}{1 + \overline{N}_2}, \qquad \overline{N}_2 = \frac{K_2 + \alpha_2 \overline{N}_1}{1 + \overline{N}_1}, \\ \underline{N}_1 &= \frac{K_1 + \alpha_1 \underline{N}_2}{1 + \underline{N}_2}, \qquad \underline{N}_2 = \frac{K_2 + \alpha_2 \underline{N}_1}{1 + \underline{N}_1} \end{aligned} \tag{43}$$

and (43) shows that $(\overline{N}_1, \overline{N}_2)$ and $(\underline{N}_1, \underline{N}_2)$ are solutions of (11). By Lemma 2, (11) has a unique positive solution $E^*(N_1^*, N_2^*)$. Hence, we conclude that

$$\overline{N}_i = \underline{N}_i = N_i^*, \quad i = 1, 2; \tag{44}$$

that is,

$$\lim_{t \to +\infty} N_i(t) = N_i^* \quad i = 1, 2. \tag{45}$$

Thus, the unique interior equilibrium $E^*(N_1^*, N_2^*)$ is globally attractive. This completes the proof of Theorem 1. □

3. Discussion

As was pointed out in the introduction section, for Lotka-Volterra type mutualism system, conditions which ensure the globally stability of the system are not so easily verified [3, 4]. In this paper, we study the stability property of the integrodifferential model of mutualism (5). By applying the iterative technique, we obtain a set of sufficient conditions which guarantee the global attractivity of the coexistence equilibrium. Our result (Theorem 1) shows that the conditions which ensure the permanence of the system are enough to ensure the global attractivity of the system. The new finding of this paper is that we found the iterative bound of the solution, such a finding is not detected by Chen and You [13]. Based on this finding, it is possible to obtain the subtle result about the stability of the system. We mention here that we did not consider the delay in the intraspecific competition; whether such kind of delay could induce bifurcation or not is still unknown; we leave this for future study.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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