Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 659152, 6 pages
http://dx.doi.org/10.1155/2014/659152

Research Article

Multiple Periodic Solutions for Discrete Nicholson's Blowflies Type System

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Received 5 January 2014; Accepted 27 February 2014; Published 27 March 2014

Academic Editor: Samir Saker

This paper is concerned with the existence of multiple periodic solutions for discrete Nicholson's blowflies type system. By using the Leggett-Williams fixed point theorem, we obtain the existence of three nonnegative periodic solutions for discrete Nicholson's blowflies type system. In order to show that, we first establish the existence of three nonnegative periodic solutions for the n-dimensional functional difference system $y(k+1) = A(k)y(k) + f(k, y(k-\tau)), k \in \mathbb{Z}$, where $A(k)$ is not assumed to be diagonal as in some earlier results. In addition, a concrete example is also given to illustrate our results.

1. Introduction and Preliminaries

In 1954 Nicholson [1] and later in 1980 Gurney et al. [2] proposed the following delay differential equation model:

$$x'(t) = -\delta x(t) + px(t-\tau)e^{-\gamma x(t-\tau)}, \quad (1)$$

where $x(t)$ is the size of the population at time t, p is the maximum per capita daily egg production, $1/\gamma$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time.

Now, Nicholson's blowflies model and its various analogous equations have attracted more and more attention. There is large literature on this topic. Recently, the study on Nicholson's blowflies type systems has attracted much attention (cf. [3–8] and references therein). In particular, several authors have made contribution on the existence of periodic solutions for Nicholson's blowflies type systems (see, e.g., [6, 7]). In addition, discrete Nicholson's blowflies type models have been studied by several authors (see, e.g., [9–12] and references therein).

Stimulated by the above works, in this paper, we consider the following discrete Nicholson's blowflies type system:

$$\begin{aligned} x_1(k+1) &= a_{11}(k)x_1(k) \\ &\quad + a_{12}(k)x_2(k) + b(k) \\ &\quad \times [x_1(k-\tau) + x_2(k-\tau)]^m \\ &\quad \times e^{-c(k)[x_1(k-\tau)+x_2(k-\tau)]}, \\ x_2(k+1) &= a_{21}(k)x_1(k) \\ &\quad + a_{22}(k)x_2(k) + b(k) \\ &\quad \times [x_1(k-\tau) + x_2(k-\tau)]^m \\ &\quad \times e^{-c(k)[x_1(k-\tau)+x_2(k-\tau)]}, \end{aligned} \quad (2)$$

where $m > 1$ is a constant, τ is a nonnegative integer, and a_{ij}, $i, j = 1, 2$, b, and c are all N-periodic functions from $\mathbb{Z}$ to $\mathbb{R}$.

In fact, there are seldom results concerning the existence of *multiple* periodic solutions for Nicholson's blowflies type equations. It seems that the only results on this topic are due to Padhi et al. [13–15], where they established several existence theorems about multiple periodic solutions of

Nicholson's blowflies type equations. In addition, recently, several authors have investigated the existence of almost periodic solutions for Nicholson's blowflies type equations (see, e.g., [11, 16, 17] and references therein). However, to the best our knowledge, there are few results concerning the existence of multiple periodic solutions for Nicholson's blowflies type systems. That is the main motivation of this paper.

Next, let us recall the Leggett-Williams fixed point theorem, which will be used in the proof of our main results.

Let X be a Banach space. A closed convex set K in X is called a cone if the following conditions are satisfied: (i) if $x \in K$, then $\lambda x \in K$ for any $\lambda \geq 0$; (ii) if $x \in K$ and $-x \in K$, then $x = 0$.

A nonnegative continuous functional ψ is said to be concave on K if ψ is continuous and

$$\psi(\mu x + (1-\mu)y) \geq \mu\psi(x) + (1-\mu)\psi(y), \quad x, y \in K, \quad \mu \in [0,1]. \tag{3}$$

Letting c_1, c_2, and c_3 be three positive constants and letting ϕ be a nonnegative continuous functional on K, we denote

$$K_{c_1} = \{y \in K : \|y\| < c_1\}, \qquad K(\phi, c_2, c_3) = \{y \in K : c_2 \leq \phi(y), \|y\| < c_3\}. \tag{4}$$

In addition, we call that ϕ is increasing on K if $\phi(x) \geq \phi(y)$ for all $x, y \in K$ with $x - y \in K$.

Lemma 1 (see [18]). *Let K be a cone in a Banach space X, let c_4 be a positive constant, let $\Phi : \overline{K}_{c_4} \to \overline{K}_{c_4}$ be a completely continuous mapping, and let ψ be a concave nonnegative continuous functional on K with $\psi(u) \leq \|u\|$ for all $u \in \overline{K}_{c_4}$. Suppose that there exist three constants c_1, c_2, and c_3 with $0 < c_1 < c_2 < c_3 \leq c_4$ such that*

(i) *$\{u \in K(\psi, c_2, c_3) : \psi(u) > c_2\} \neq \emptyset$, and $\psi(\Phi u) > c_2$ for all $u \in K(\psi, c_2, c_3)$;*

(ii) *$\|\Phi u\| < c_1$ for all $u \in \overline{K}_{c_1}$;*

(iii) *$\psi(\Phi u) > c_2$ for all $u \in K(\psi, c_2, c_4)$ with $\|\Phi u\| > c_3$.*

Then Φ has at least three fixed points u_1, u_2, and u_3 in $\overline{K}_{c_4}$. Furthermore, $\|u_1\| \leq c_1 < \|u_2\|$, and $\psi(u_2) < c_2 < \psi(u_3)$.

Throughout the rest of this paper, we denote by $\mathbb{Z}$ the set of all integers, by $\mathbb{R}$ the set of all real numbers, and by $l_N^\infty(\mathbb{Z}, \mathbb{R}^n)$ the space of all N-periodic functions $x : \mathbb{Z} \to \mathbb{R}^n$, where N is a fixed positive integer. It is easy to see that $l_N^\infty(\mathbb{Z}, \mathbb{R}^n)$ is a Banach space under the norm

$$\|x\| = \max_{1 \leq k \leq N} \max_{1 \leq i \leq n} |x_i(k)|, \tag{5}$$

where $x = (x_1, x_2, \ldots, x_n)^T$. In addition, we denote

$$\mathbb{R}_+^n := \{(x_1, x_2, \ldots, x_n) : x_1, x_2, \ldots, x_n \geq 0\}. \tag{6}$$

2. Main Results

To study the existence of multiple periodic solutions for system (2), we first consider the following more general n-dimensional functional difference system:

$$y(k+1) = A(k)y(k) + f(k, y(k-\tau)), \quad k \in \mathbb{Z}, \tag{7}$$

where, for every $k \in \mathbb{Z}$, $A(k)$ is N-periodic and nonsingular $n \times n$ matrix, and $f = (f_1, \ldots, f_n)^T : \mathbb{Z} \times \mathbb{R}^n \to \mathbb{R}^n$ is N-periodic in the first argument and continuous in the second argument.

To note that the existence of periodic solutions for system (7) and its variants had been of great interest for many authors (see, e.g., [19–25] and references therein) is needed. However, in some earlier works (see, e.g., [21]) on the existence of periodic solutions for system (7), the matrix $A(k)$ is assumed to be diagonal. In this paper, we will remove this restrictive condition by utilizing an idea in [22], where the authors studied the existence of periodic solutions for a class of nonlinear neutral systems of differential equations.

Let $\Phi(0) = I$,

$$\begin{gathered}
\Phi(k) = \prod_{i=0}^{k-1} A(i) = A(k-1) \cdots A(0), \quad k \geq 1, \\
\Phi(k) = \prod_{i=k}^{-1} [A(i)]^{-1} = [A(k)]^{-1} \cdots [A(-1)]^{-1}, \quad k \leq -1, \\
G(k,s) = \Phi(k)\left[\Phi^{-1}(N) - I\right]^{-1} \Phi^{-1}(s+1), \quad k \in \mathbb{Z}, \quad k \leq s \leq k+N-1.
\end{gathered} \tag{8}$$

We first present some basic results about $\Phi(k)$ and $G(k,s)$.

Lemma 2. *For all $k, s \in \mathbb{Z}$ with $k \leq s \leq k + N - 1$, the following assertions hold:*

(i) $\Phi(k+1) = A(k)\Phi(k)$,

(ii) $\Phi(k+N) = \Phi(k)\Phi(N)$,

(iii) $G(k+1, s) = A(k)G(k, s)$,

(iv) $G(k+N, s+N) = G(k, s)$.

Proof. One can show (i) and (ii) by some direct calculations and noting that $A(k+N) = A(k)$. So we omit the details. In addition, the assertion (iii) follows from the assertion (i) and the assertion (iv) follows from the assertion (ii). □

By using Lemma 2, we can get the following result.

Lemma 3. *A function $y : \mathbb{Z} \to \mathbb{R}^n$ is a N-periodic solution of system (7) if and only if y is a N-periodic function satisfying*

$$y(k) = \sum_{s=k}^{k+N-1} G(k,s) f(s, y(s-\tau)), \quad k \in \mathbb{Z}. \tag{9}$$

Proof. Sufficiency. Assume that $y : \mathbb{Z} \to \mathbb{R}^n$ is a N-periodic function satisfying (9); that is,

$$y(k) = \sum_{s=k}^{k+N-1} G(k,s) f(s, y(s-\tau)), \quad k \in \mathbb{Z}. \tag{10}$$

Then, we have

$$\begin{aligned} & y(k+1) \\ &\quad = \sum_{s=k+1}^{k+N} G(k+1,s) f(s, y(s-\tau)) \\ &\quad = \sum_{s=k+1}^{k+N-1} G(k+1,s) f(s, y(s-\tau)) \\ &\qquad + G(k+1,k+N) f(k+N, y(k+N-\tau)) \\ &\quad = \sum_{s=k+1}^{k+N-1} A(k) G(k,s) f(s, y(s-\tau)) \\ &\qquad + G(k+1,k+N) f(k, y(k-\tau)) \\ &\quad = \sum_{s=k}^{k+N-1} A(k) G(k,s) f(s, y(s-\tau)) \\ &\qquad - A(k) G(k,k) f(k, y(k-\tau)) \\ &\qquad + G(k+1,k+N) f(k, y(k-\tau)) \\ &\quad = A(k) y(k) - A(k) G(k,k) f(k, y(k-\tau)) \\ &\qquad + G(k+1,k+N) f(k, y(k-\tau)) \\ &\quad = A(k) y(k) + f(k, y(k-\tau)), \end{aligned} \tag{11}$$

where

$$\begin{aligned} & G(k+1,k+N) - A(k) G(k,k) \\ &\quad = \Phi(k+1) \left[\Phi^{-1}(N) - I\right]^{-1} \Phi^{-1}(k+N+1) \\ &\qquad - A(k) \Phi(k) \left[\Phi^{-1}(N) - I\right]^{-1} \Phi^{-1}(k+1) \\ &\quad = \Phi(k+1) \left[\Phi^{-1}(N) - I\right]^{-1} \Phi^{-1}(N) \Phi^{-1}(k+1) \\ &\qquad - \Phi(k+1) \left[\Phi^{-1}(N) - I\right]^{-1} \Phi^{-1}(k+1) \\ &\quad = \Phi(k+1) \Phi^{-1}(k+1) = I. \end{aligned} \tag{12}$$

Thus, we conclude that y is a N-periodic solution of system (7).
Necessity. Let $y : \mathbb{Z} \to \mathbb{R}^n$ be a N-periodic solution of system (7). Then, we have

$$y(s+1) = A(s) y(s) + f(s, y(s-\tau)), \quad s \in \mathbb{Z}, \tag{13}$$

which yields

$$\begin{aligned} & \Phi^{-1}(s+1) y(s+1) - \Phi^{-1}(s) y(s) \\ &\quad = \Phi^{-1}(s+1) \left[A(s) y(s) + f(s, y(s-\tau))\right] \\ &\qquad - \Phi^{-1}(s) y(s) = \Phi^{-1}(s+1) f(s, y(s-\tau)), \\ &\qquad\qquad s \in \mathbb{Z}. \end{aligned} \tag{14}$$

For all $l \geq k$, we have

$$\begin{aligned} & \Phi^{-1}(l+1) y(l+1) - \Phi^{-1}(k) y(k) \\ &\quad = \sum_{s=k}^{l} \left[\Phi^{-1}(s+1) y(s+1) - \Phi^{-1}(s) y(s)\right] \\ &\quad = \sum_{s=k}^{l} \Phi^{-1}(s+1) f(s, y(s-\tau)), \end{aligned} \tag{15}$$

which yields

$$\begin{aligned} & \Phi^{-1}(l+1) y(l+1) \\ &\quad = \Phi^{-1}(k) y(k) + \sum_{s=k}^{l} \Phi^{-1}(s+1) f(s, y(s-\tau)). \end{aligned} \tag{16}$$

Letting $l = k+N-1$ and noting that y is N-periodic, we get

$$\begin{aligned} & \Phi^{-1}(k+N) y(k) \\ &\quad = \Phi^{-1}(k+N) y(k+N) \\ &\quad = \Phi^{-1}(k) y(k) + \sum_{s=k}^{k+N-1} \Phi^{-1}(s+1) f(s, y(s-\tau)). \end{aligned} \tag{17}$$

Noting that

$$\Phi^{-1}(k+N) - \Phi^{-1}(k) = \left[\Phi^{-1}(N) - I\right] \Phi^{-1}(k), \tag{18}$$

we conclude

$$\begin{aligned} y(k) &= \Phi(k) \left[\Phi^{-1}(N) - I\right]^{-1} \\ &\qquad \times \sum_{s=k}^{k+N-1} \Phi^{-1}(s+1) f(s, y(s-\tau)) \\ &= \sum_{s=k}^{k+N-1} G(k,s) f(s, y(s-\tau)). \end{aligned} \tag{19}$$

That is, (9) holds. This completes the proof. □

Let

$$\begin{aligned} G(k,s) &= \left[G_{ij}(k,s)\right], \\ p &= \min_{1\leq i\leq n} \min_{1\leq k\leq N} \min_{k\leq s\leq k+N-1} \sum_{j=1}^{n} G_{ij}(k,s), \\ q &= \max_{1\leq i\leq n} \max_{1\leq k\leq N} \max_{k\leq s\leq k+N-1} \sum_{j=1}^{n} G_{ij}(k,s). \end{aligned} \tag{20}$$

Now, we introduce a set

$$K = \{x \in l_N^\infty(\mathbb{Z}, \mathbb{R}^n) : x_i(k) \ge 0, \quad k = 1, 2, \dots, N, i = 1, 2, \dots, n\}. \tag{21}$$

It is not difficult to verify that K is a cone in $l_N^\infty(\mathbb{Z}, \mathbb{R}^n)$. Finally, we define an operator Φ on K by

$$(\Phi x)(k) = \sum_{s=k}^{k+N-1} G(k,s) f(s, x(s-\tau)), \quad x \in K, \quad k \in \mathbb{Z}. \tag{22}$$

Theorem 4. *Assume that $f_1 = f_2 = \cdots = f_n$ and the following assumptions hold.*

(H0) $q > p > 0$, $f_1(s,x) \ge 0$ for all $s \in \mathbb{Z}$ and $x \in \mathbb{R}_+^n$, and $\sum_{j=1}^n G_{ij}(k,s) \ge 0$ for all $k \in \mathbb{Z}$, $k \le s \le k+N-1$, and $i = 1, 2, \dots, n$.

(H1) There exist two constants $c_4 > c_1 > 0$ such that

$$q \cdot \sum_{s=1}^N f_1(s,x) < c_1 \quad \text{for } x \in \mathbb{R}_+^n \text{ with } \|x\| \le c_1, \qquad q \cdot \sum_{s=1}^N f_1(s,x) \le c_4 \quad \text{for } x \in \mathbb{R}_+^n \text{ with } \|x\| \le c_4. \tag{23}$$

(H2) There exists a constant $c_2 \in (c_1, c_4)$ such that $qc_2 \le pc_4$, and

$$p \cdot \sum_{s=1}^N f_1(s,x) > c_2 \quad \text{for } x \in \mathbb{R}_+^n \text{ with } \|x\| < \frac{q}{p} c_2, \quad \sum_{i=1}^n x_i \ge nc_2. \tag{24}$$

Then system (7) has at least three nonnegative N-periodic solutions.

Proof. Firstly, by (H0) and noting that $G(k+N, s+N) = G(k,s)$, Φ is an operator from K to K. Secondly, noting that f is continuous for the second argument, by similar proof to [21, Lemma 2.5], one can show that $\Phi : K \to K$ is completely continuous.

Let

$$\psi(x) = \min_{1\le k\le N} \frac{\sum_{i=1}^n x_i(k)}{n}, \quad x \in K. \tag{25}$$

It is easy to see that ψ is a concave nonnegative continuous functional on K and $\psi(x) \le \|x\|$.

Now, we show that Φ maps $\overline{K}_{c_4}$ into $\overline{K}_{c_4}$. For every $x \in \overline{K}_{c_4}$, we have $x(s-\tau) \in \mathbb{R}_+^n$ and $\|x(s-\tau)\| \le c_4$ for all $s \in \mathbb{Z}$. Then, by (H1), we have

$$\begin{aligned} \|\Phi x\| &= \max_{1\le k\le N} \max_{1\le i\le n} \sum_{s=k}^{k+N-1} \sum_{j=1}^n G_{ij}(k,s) f_j(s, x(s-\tau)) \\ &\le q \cdot \sum_{s=1}^N f_1(s, x(s-\tau)) \le c_4. \end{aligned} \tag{26}$$

Similarly, for every $x \in \overline{K}_{c_1}$, it follows from (H1) that

$$\|\Phi x\| \le q \cdot \sum_{s=1}^N f_1(s, x(s-\tau)) < c_1. \tag{27}$$

That is, condition (ii) of Lemma 1 holds.

Let $c_3 = (q/p)c_2$. Next, let us verify condition (i) of Lemma 1. It is easy to see that the set

$$\{x \in K(\psi, c_2, c_3) : \psi(x) > c_2\} \ne \emptyset. \tag{28}$$

In addition, for every $x \in K(\psi, c_2, c_3)$, we have $x(s-\tau) \in \mathbb{R}_+^n$, $\|x(s-\tau)\| < c_3 = (q/p)c_2$, and $\sum_{i=1}^n x_i(s-\tau) \ge nc_2$ for all $s \in \mathbb{Z}$. Then, by (H2), we get

$$\begin{aligned} \psi(\Phi x) &= \frac{1}{n} \cdot \min_{1\le k\le N} \sum_{i=1}^n \sum_{s=k}^{k+N-1} \sum_{j=1}^n G_{ij}(k,s) f_j(s, x(s-\tau)) \\ &\ge p \cdot \sum_{s=1}^N f_1(s, x(s-\tau)) > c_2 \end{aligned} \tag{29}$$

which means that condition (i) of Lemma 1 holds.

It remains to verify that condition (iii) of Lemma 1 holds. Let $x \in K(\psi, c_2, c_4)$ with $\|\Phi x\| > c_3$; we have $c_2 \le \|x\| < c_4$ and

$$q \cdot \sum_{s=1}^N f_1(s, x(s-\tau)) \ge \|\Phi x\| > c_3, \tag{30}$$

which yields

$$\sum_{s=1}^N f_1(s, x(s-\tau)) > \frac{c_3}{q} = \frac{c_2}{p}. \tag{31}$$

Then, we have

$$\psi(\Phi x) \ge p \cdot \sum_{s=1}^N f_1(s, x(s-\tau)) > c_2. \tag{32}$$

Then, by Lemma 1, we know that Φ has at least three fixed points in $\overline{K}_{c_4}$. Then, it follows from Lemma 3 that system (7) has at least three nonnegative N-periodic solutions. □

Now, we apply Theorem 4 to Nicholson's blowflies system (2). Let $n = 2$,

$$A(k) = \begin{pmatrix} a_{11}(k) & a_{12}(k) \\ a_{21}(k) & a_{22}(k) \end{pmatrix},$$
$$f_1(k,x) = f_2(k,x) = b(k)\left[x_1 + x_2\right]^m e^{-c(k)[x_1+x_2]}, \tag{33}$$

and let $\Phi(k)$, $G(k,s)$, p, q, and K be as in Theorem 4.

Corollary 5. *Assume that $q > p > 0$, and $\sum_{j=1}^{2} G_{ij}(k,s)$ ($i = 1,2$), $b(k)$, and $c(k)$ are all nonnegative for $k \in \mathbb{Z}$ and $k \le s \le k+N-1$. Then the system (2) has at least three nonnegative N-periodic solutions provided that $c^+ := \max_{1\le s\le N} c(s) \ge c^- := \min_{1\le s\le N} c(s) > 0$, and*

$$p \cdot 2^m \cdot \sum_{s=1}^{N} b(s) > e^{m-1} \cdot \left[\frac{2c^+ q}{p(m-1)}\right]^{m-1}. \tag{34}$$

Proof. We only need to verify that all the assumptions of Theorem 4 are satisfied. Firstly, it is easy to see that (H0) holds. Let

$$c_2 = \frac{p(m-1)}{2c^+ q}. \tag{35}$$

Secondly, let us check (H1). In fact, one can choose sufficiently small $c_1 \in (0, c_2)$ such that, for all $x \in \mathbb{R}_+^n$ with $\|x\| \le c_1$, there holds

$$\begin{aligned} q \cdot \sum_{s=1}^{N} f_1(s,x) &= q \cdot \sum_{s=1}^{N} b(s)\left[x_1 + x_2\right]^m e^{-c(s)[x_1+x_2]} \\ &\le q \cdot \sum_{s=1}^{N} b(s) \cdot 2^m \cdot \|x\|^m \\ &= \left(2^m q \cdot \sum_{s=1}^{N} b(s)\right) \cdot \|x\|^m < \|x\| \le c_1. \end{aligned} \tag{36}$$

In addition, for all $x \in \mathbb{R}_+^n$, we have

$$\begin{aligned} q \cdot \sum_{s=1}^{N} f_1(s,x) &\le q \cdot \sum_{s=1}^{N} b(s)\left[x_1 + x_2\right]^m e^{-c^-[x_1+x_2]} \\ &\le q \cdot \sum_{s=1}^{N} b(s) \cdot \left(\frac{m}{c^-}\right)^m e^{-m}. \end{aligned} \tag{37}$$

So, letting

$$c_4 = \max\left\{\frac{qc_2}{p}, q \cdot \sum_{s=1}^{N} b(s) \cdot \left(\frac{m}{c^-}\right)^m e^{-m}\right\}, \tag{38}$$

we conclude that (H1) holds.

It remains to verify (H2). For all $x \in \mathbb{R}_+^n$ with $\|x\| < (q/p)c_2$ and $\sum_{i=1}^{2} x_i \ge 2c_2$, by using (34), we have

$$\begin{aligned} p \cdot \sum_{s=1}^{N} f_1(s,x) &= p \cdot \sum_{s=1}^{N} b(s)\left[x_1 + x_2\right]^m e^{-c(s)[x_1+x_2]} \\ &\ge p \cdot \sum_{s=1}^{N} b(s) \cdot 2^m c_2^m \cdot e^{-(2c^+ q/p)c_2} \\ &= \left(p \cdot 2^m \cdot \sum_{s=1}^{N} b(s)\right) \cdot c_2^{m-1} \cdot e^{-(2c^+ q/p)c_2} \cdot c_2 \\ &= \left(p \cdot 2^m \cdot \sum_{s=1}^{N} b(s) \cdot e^{-(m-1)} \cdot \left[\frac{p(m-1)}{2c^+ q}\right]^{m-1}\right) \cdot c_2 \\ &> c_2. \end{aligned} \tag{39}$$

This completes the proof. □

Next, we give a concrete example for Nicholson's blowflies type system (2).

Example 6. Let $m = N = 2$, $\tau = 1$, $b(k) = 100 + \sin^2(\pi k/2)$, $c(k) = 1 + \cos^2(\pi k/2)$, and

$$A(0) = \begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix}, \qquad A(1) = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & 0 \end{pmatrix}. \tag{40}$$

By a direct calculation, we can get

$$\begin{aligned} &G(1,1) = \begin{pmatrix} 0 & \frac{3}{8} \\ \frac{2}{3} & 0 \end{pmatrix}, \qquad G(1,2) = \begin{pmatrix} \frac{9}{8} & 0 \\ 0 & \frac{4}{3} \end{pmatrix}, \\ &G(2,2) = \begin{pmatrix} 0 & \frac{2}{3} \\ \frac{3}{8} & 0 \end{pmatrix}, \qquad G(2,3) = \begin{pmatrix} \frac{4}{3} & 0 \\ 0 & \frac{9}{8} \end{pmatrix}. \end{aligned} \tag{41}$$

Then, we have $p = 3/8$ and $q = 4/3$. In addition, we have $c^+ = 2 > c^- = 1 > 0$ and

$$\begin{aligned} p \cdot 2^m \cdot \sum_{s=1}^{N} b(s) &= \frac{3}{8} \cdot 4 \cdot 201 \\ &= \frac{603}{2} > \frac{128}{9} e = e^{m-1} \cdot \left[\frac{2c^+ q}{p(m-1)}\right]^{m-1}. \end{aligned} \tag{42}$$

So, by Corollary 5, we know that system (2) has at least three nonnegative 2-periodic solutions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors are grateful to the referees for valuable suggestions and comments. Hui-Sheng Ding acknowledges support from the NSF of China (11101192), the Program for Cultivating Young Scientist of Jiangxi Province (20133BCB23009), and the NSF of Jiangxi Province.

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