Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 248061, 9 pages
http://dx.doi.org/10.1155/2014/248061

Research Article

Improved Stability Criteria for Markovian Jump Systems with Time-Varying Delays

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Received 16 May 2014; Accepted 9 July 2014; Published 21 July 2014

Academic Editor: Yonghui Xia

The delay-dependent stochastic stability problem of Markovian jump systems with time-varying delays is investigated in this paper. Though the Lyapunov-Krasovskii functional is general and simple, less conservative results are derived by using the convex combination method, improved Wirtinger's integral inequality, and a slack condition on Lyapunov matrix. The obtained results are formulated in terms of linear matrix inequalities (LMIs). Numerical examples are provided to verify the effectiveness and superiority of the presented results.

1. Introduction

Markovian jump systems are a special class of stochastic hybrid systems. Many dynamical systems subject to random abrupt variations, such as mechanical systems, economics, and systems with human operators, can be modeled by Markovian jump systems [1]. Due to their extensive applications in many files, the analysis and synthesis of Markovian jump systems have received much research attention and lots of significant results have been reported; see, for example, [2–8] and the references therein.

Time delay is an inherent characteristic of many dynamic systems such as networked control systems, industrial systems, and process control systems. The systems with or without time delays are convergent when time delays are close to zero; otherwise, they may be divergent. In other words, time delays can degrade the performance of systems designed without considering the delays and can even destabilize the systems. During the past few decades, considerable attention has been paid to the stability analysis of time-delay systems [9–16]. The existing stability criteria for linear systems can be classified into two types: delay-independent ones which are applicable to delays of arbitrary size and delay-dependent ones which include information on the size of delays. In general, delay-dependent stability criteria are less conservative than delay-independent ones especially when the size of the delay is small. Thus, considerable attention has been paid to the delay-dependent stability criteria; see [11–23], for example. As for delay-dependent stability, many methods have been taken for deriving stability criteria, such as free-weighting matrices methods [11], model transformation techniques [12, 13], convex combination methods [14–17], delay decomposition approaches [18], multiple integral approaches [19], and input-output approaches [20]. Recently, the bounding techniques of the cross terms and integral terms in the derivatives of the Lyapunov-Krasovskii functional are widely investigated, such as improved Jensen's integral inequality [21], reciprocally convex approach [22, 23], and improved Wirtinger's integral inequality [24]. Some less conservative stability results have been derived by using the above techniques.

In this paper, we develop some new stability criteria by using an improved Wirtinger's integral inequality and the convex combination method to deal with the cross terms and integral terms in the derivatives of the Lyapunov-Krasovskii functional. In addition, the positive definiteness of some Lyapunov matrix is not required. The obtained results can be applied to both slow and fast time-varying delays. The numerical examples demonstrate the effectiveness and superiority of the presented results.

Notation. Throughout this paper, (Ω, F, P) is a probability space, Ω is the sample space, F is the σ-algebra of the sample space, and P is the probability measure on F. $\mathbb{E}\{\cdot\}$ refers to the expectation operator with respect to some probability measure P. $A > 0$ (<0) means A is a symmetric positive (negative) definite matrix and A^{-1} denotes the inverse of matrix A. A^T represents the transpose of A. $\mathrm{Sym}(M)$ stands for $M + M^T$. The symbol $*$ in LMIs denotes the symmetric term of the matrix. $\mathrm{col}\{X, Y\}$ represents a column vector formed by X and Y. Identity matrix, of appropriate dimensions, will be denoted by I. $\mathrm{diag}(\cdot,\cdot)$ denotes a diagonal matrix. $\Omega(i, j)$ means the element in the ith row and jth column of the block matrix Ω.

2. Problem Statement

Fix a probability space (Ω, F, P) and consider the following Markovian jump systems:

$$\begin{aligned} \dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-d(t)), \\ x(t) &= \phi(t), \quad t \in [-d_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $\phi(t)$ is a compatible vector-valued initial function defined on $[-d_2, 0]$. $\{r_t, t \geqslant 0\}$ is a continuous-time Markovian process taking values in a finite space $S = \{1, 2, \ldots, N\}$; $A(r_t)$ and $A_d(r_t)(r_t = i,\ i \in S)$ are real constant matrices with appropriate dimensions which depend on r_t. The time delay $d(t)$ satisfies

$$0 < d_1 \leqslant d(t) \leqslant d_2, \quad \dot{d}(t) \leqslant \mu. \tag{2}$$

The evolution of the Markovian process $\{r_t, t \geqslant 0\}$ is governed by the following transition probability:

$$\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & j \neq i, \\ 1 + \pi_{ii}\Delta + o(\Delta), & j = i, \end{cases} \tag{3}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$; $\pi_{ij} \geqslant 0$ for $i \neq j$ is the transition probability from mode i at time t to mode j at time $t + \Delta$ and $\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij}$.

For simplicity, when $r_t = i$, $i \in S$, the matrices $A(r_t)$ and $A_d(r_t)$ are denoted by A_i and A_{di}, respectively.

The following definition and lemmas are needed in the proof of our main results.

Definition 1 (see [2]). System (1) is stochastically stable, if, for any initial state (x_0, r_0), the following relation holds for any initial condition (x_0, r_0):

$$\lim_{t\to+\infty} \mathbb{E}\left\{\|x(t)\|^2 \mid x_0, r_0\right\} = 0. \tag{4}$$

Lemma 2 (see [22]). *Let $f_1, f_2, \ldots, f_n : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies*

$$\min_{\{\alpha_i \mid \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t) \tag{5}$$

subject to

$$\left\{ g_{i,j} : \mathbb{R}^m \longrightarrow \mathbb{R}, g_{j,i} \triangleq g_{i,j}, \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geqslant 0 \right\}. \tag{6}$$

Lemma 3 (see [24]). *If $\omega(y)$ is a differentiable function in $[a, b] \to \mathbb{R}^n$. Then for any given symmetric positive definite matrix $R > 0$, the following inequality holds:*

$$\int_a^b \dot{\omega}^T(t) R \dot{\omega}(t)\, dt \geqslant \frac{1}{b-a} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}^T \widetilde{R} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \tag{7}$$

where $\nu_1 = \omega(b) - \omega(a)$, $\nu_2 = \omega(b) + \omega(a) - (2/(b-a)) \int_a^b \omega(t)dt$, and $\widetilde{R} = \mathrm{diag}(R, 3R)$.

3. Improved Stability Criterion

In this section, we will present an improved stochastic stability criterion in terms of LMIs by using Lyapunov-Krasovskii functional method and convex combination technique.

Before stating the main results, some notations are given. Let

$$\begin{aligned} d_{12} &= d_2 - d_1, \\ \zeta(t) &= \mathrm{col}\left\{ x(t), \int_{t-d_1}^{t} x(s)\,\mathrm{d}s, \int_{t-d_2}^{t-d_1} x(s)\,\mathrm{d}s \right\}, \\ \xi(t) &= \mathrm{col}\Bigg\{ x(t), x(t-d(t)), x(t-d_1), x(t-d_2), \\ &\quad \frac{1}{d_1}\int_{t-d_1}^{t} x(s)\,\mathrm{d}s, \frac{1}{d(t)-d_1}\int_{t-d(t)}^{t-d_1} x(s)\,\mathrm{d}s, \\ &\quad \frac{1}{d_2-d(t)}\int_{t-d_2}^{t-d(t)} x(s)\,\mathrm{d}s \Bigg\}, \\ \mathscr{P}_i &= \begin{bmatrix} P_{11i} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix}, \quad (i \in S), \\ \Gamma_1^T &= \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_1 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (d(t)-d_1)I & (d_2-d(t))I \end{bmatrix}, \\ X &= \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & P_{22} \end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix} A_i & A_{di} & 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -I & 0 & 0 & 0 \end{bmatrix}, \quad (i \in S). \end{aligned} \tag{8}$$

Theorem 4. *Given scalars $0 < d_1 < d_2$ and μ, the time-varying delay system (1) is stochastically stable, if there exist matrix $\mathscr{P}_i \in \mathbb{R}^{3n\times 3n}$; symmetric positive definite matrices $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{n\times n}$, $Q_3 \in \mathbb{R}^{n\times n}$, and $M \in \mathbb{R}^{n\times n}$; and any matrices $X \in \mathbb{R}^{n\times n}$, $N_l \in \mathbb{R}^{n\times n}$, and $G_l \in \mathbb{R}^{n\times n}$ $(l = 1, 2)$ such that, for all $i \in S$,*

$$\Xi_1 = \begin{bmatrix} R_2 - M & 0 & X_{11} & X_{12} \\ * & 3(R_2 - M) & X_{21} & X_{22} \\ * & * & R_2 - M & 0 \\ * & * & 0 & 3(R_2 - M) \end{bmatrix} > 0, \quad (9)$$

$$\Xi_{2i} = \begin{bmatrix} P_{11i} + 2d_1 R_1 + \frac{2d_{12}^2}{d_1 + d_2} R_2 & P_{12} - 2R_1 & P_{13} - \frac{2d_{12}}{d_1 + d_2} R_2 \\ * & P_{22} + \frac{1}{d_1}(Q_1 + 2R_1) & P_{23} \\ * & * & P_{33} + \frac{1}{d_{12}} Q_2 + \frac{2}{d_1 + d_2} R_2 \end{bmatrix} > 0, \quad (10)$$

$$\Xi_{3i} = \begin{bmatrix} [\Omega_1]_{5n\times 5n} & [\Omega_2]_{5n\times n} & [\Omega_3]_{5n\times n} & [\Omega_4]_{5n\times n} & [\Omega_5]_{5n\times n} & [\Omega_6]_{5n\times n} \\ * & -12(R_2 - M) & -4X_{22} & 0 & 0 & 0 \\ * & * & -12(R_2 - M) & 0 & 0 & 0 \\ * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & -M \end{bmatrix} < 0, \quad (11)$$

$$\Xi_{4i} = \begin{bmatrix} [\Omega_1]_{5n\times 5n} & [\widehat{\Omega}_2]_{5n\times n} & [\widehat{\Omega}_3]_{5n\times n} & [\Omega_4]_{5n\times n} & [\Omega_5]_{5n\times n} & [\widehat{\Omega}_6]_{5n\times n} \\ * & -12(R_2 - M) & -4X_{22} & 0 & 0 & 0 \\ * & * & -12(R_2 - M) & 0 & 0 & 0 \\ * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & -M \end{bmatrix} < 0, \quad (12)$$

where

$$\Omega_1(1,1) = \text{Sym}(P_{11i}A_i + P_{12}) + \sum_{j=1}^{N} \pi_{ij} P_{11j} + Q_1 - 4R_1,$$

$$\Omega_1(1,2) = P_{11i}A_{di},$$

$$\Omega_1(1,3) = -2R_1 - P_{12} + P_{13},$$

$$\Omega_1(1,4) = -P_{13}, \qquad \Omega_1(1,5) = 6R_1 + d_1\left(A_i^T P_{12} + P_{22}^T\right),$$

$$\Omega_1(2,2) = -(1-\mu)Q_3 - 8R_2 + 8M + \text{Sym}(X_{11} + X_{12} - X_{21} - X_{22} - N_1 + G_1),$$

$$\Omega_1(2,3) = -X_{11}^T - X_{12}^T - X_{21}^T - X_{22}^T - 2R_2 + N_1 - N_2^T + G_2^T + 2M,$$

$$\Omega_1(2,4) = -2R_2 - X_{11} + X_{12} + X_{21} - X_{22} - N_3^T - G_1^T + G_3^T + 2M,$$

$$\Omega_1(2,5) = d_1 A_{di}^T P_{12}$$

$$\Omega_1(3,3) = -4R_1 - Q_1 + Q_2 + Q_3 - 4R_2 + 4M + \text{Sym}(N_2),$$

$$\Omega_1(3,4) = X_{11} - X_{12} + X_{21} - X_{22} + N_3 - G_2,$$

$$\Omega_1(3,5) = 6R_1 + d_1\left(P_{23}^T - P_{22}^T\right),$$

$$\Omega_1(4,4) = -Q_2 - 4R_2 + 4M - \text{Sym}(G_3),$$

$$\Omega_1(4,5) = -d_1 P_{23}^T, \qquad \Omega_1(5,5) = -12R_1,$$

$$\Omega_2(1,1) = d_{12}\left(A_i^T P_{13} + P_{23}\right),$$

$$\Omega_2(2,1) = d_{12} A_{di}^T P_{13} + 2X_{21}^T + 2X_{22}^T + 6R_6 - 6M,$$

$$\Omega_2(3,1) = -d_{12}\left(P_{23} - P_{33}^T\right) + 6R_6 - 6M,$$

$$\Omega_2(4,1) = -d_{12} P_{33}^T - 2X_{21}^T + 2X_{22}^T, \qquad \Omega_2(5,1) = 0,$$

$$\Omega_3(1,1) = 0, \qquad \Omega_3(2,1) = 6R_2 - 6M - 2X_{12} + 2X_{22},$$

$$\Omega_3(3,1) = 2X_{12} + 2X_{22},$$

$$\Omega_3(4,1) = 6R_2 - 6M, \qquad \Omega_3(5,1) = 0,$$

$$\Omega_4(1,1) = d_1 A_i^T R_1, \qquad \Omega_4(2,1) = d_1 A_{di}^T R_1,$$

$$\Omega_4(3,1) = \Omega_4(4,1) = \Omega_4(5,1) = 0,$$

$$\Omega_5(1,1) = d_{12} A_i^T R_2, \qquad \Omega_5(2,1) = d_{12} A_{di}^T R_2,$$

$$\Omega_5(3,1) = \Omega_5(4,1) = \Omega_5(5,1) = 0,$$

$$\Omega_6(1,1) = 0, \qquad \Omega_6(2,1) = N_1, \qquad \Omega_6(3,1) = N_2,$$

$$\Omega_6(4,1) = N_3, \qquad \Omega_6(5,1) = 0,$$

$$\widehat{\Omega}_2(1,1) = 0, \qquad \widehat{\Omega}_2(2,1) = 2X_{21}^T + 2X_{22}^T + 6R_6 - 6M,$$

$$\widehat{\Omega}_2(3,1) = 6R_6 - 6M,$$

$$\widehat{\Omega}_2(4,1) = -2X_{21}^T + 2X_{22}^T, \qquad \widehat{\Omega}_2(5,1) = 0,$$

$$\widehat{\Omega}_3(1,1) = d_{12}\left(A_i^T P_{13} + P_{23}\right),$$
$$\widehat{\Omega}_3(2,1) = d_{12}A_{di}^T P_{13} + 6R_2 - 6M - 2X_{12} + 2X_{22},$$
$$\widehat{\Omega}_3(3,1) = -d_{12}\left(P_{23} - P_{33}^T\right) + 2X_{12} + 2X_{22},$$
$$\widehat{\Omega}_3(4,1) = -d_{12}P_{33}^T + 6R_2 - 6M, \qquad \widehat{\Omega}_3(5,1) = 0,$$
$$\widehat{\Omega}_6(1,1) = 0, \qquad \widehat{\Omega}_6(2,1) = G_1, \qquad \Omega_6(3,1) = G_2,$$
$$\widehat{\Omega}_6(4,1) = G_3, \qquad \widehat{\Omega}_6(5,1) = 0. \tag{13}$$

Proof. Consider the Lyapunov-Krasovskii functional given by

$$V(x_t, i) = V_1(x_t, i) + V_2(x_t, i) + V_3(x_t, i), \tag{14}$$

where

$$\begin{aligned} V_1(x_t, i) &= \zeta^T(t)\mathscr{P}(r_t)\zeta(t), \\ V_2(x_t, i) &= \int_{t-d_1}^{t} x^T(s)Q_1 x(s)\,ds \\ &\quad + \int_{t-d_2}^{t-d_1} x^T(s)Q_2 x(s)\,ds \\ &\quad + \int_{t-d(t)}^{t-d_1} x^T(s)Q_3 x(s)\,ds, \\ V_3(x_t, i) &= d_1\int_{-d_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\,d\theta \\ &\quad + d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^T(s)R_2\dot{x}(s)\,ds\,d\theta. \end{aligned} \tag{15}$$

We first show, for some $\epsilon > 0$, the Lyapunov-Krasovskii functional condition $V(x_t, i) \geqslant \epsilon\|x_t\|^2$ for any initial condition (x_0, r_0). Note that $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, and $R_2 > 0$; it follows easily from Jensen's inequality that

$$\begin{aligned} V(x_t, i) &\geqslant \zeta^T(t)\mathscr{P}_i\zeta(t) + \frac{1}{d_1}\int_{t-d_1}^{t} x^T(s)\,ds Q_1 \\ &\quad \times \int_{t-d_1}^{t} x(s)\,ds + \frac{1}{d_{12}}\int_{t-d_2}^{t-d_1} x^T(s)\,ds Q_2 \\ &\quad \times \int_{t-d_2}^{t-d_1} x(s)\,ds + \int_{t-d(t)}^{t-d_1} x^T(s)Q_3 x(s)\,ds \\ &\quad + \frac{2}{d_1}\left(d_1 x^T(t) - \int_{t-d_1}^{t} x^T(s)\,ds\right) \\ &\quad \times R_1\left(d_1 x(t) - \int_{t-d_1}^{t} x(s)\,ds\right) \\ &\quad + \frac{2d_{12}}{d_2^2 - d_1^2}\left(d_{12}x^T(t) - \int_{t-d_2}^{t-d_1} x^T(s)\,ds\right) \\ &\quad \times R_2\left(d_{12}x(t) - \int_{t-d_2}^{t-d_1} x(s)\,ds\right) \\ &= \zeta^T(t)\Xi_{2i}\zeta(t) + \int_{t-d(t)}^{t-d_1} x^T(s)Q_3 x(s)\,ds. \end{aligned} \tag{16}$$

Thus, if $Q_3 > 0$ and $\Xi_{2i} > 0$, then there exists a sufficiently small $\epsilon > 0$ such that $V(x_t, i) \geqslant \epsilon\|x_t\|^2$.

We next show that $\mathscr{L}V(x_t, i) \leqslant -\epsilon\|x(t)\|^2$ for the sufficiently small ϵ. Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{x_t, r_t\}$. Calculating the difference of $V(x_t, i)$ along the trajectories of (1), we have

$$\begin{aligned} \mathscr{L}V(x_t, i) &= 2\zeta^T(t)\mathscr{P}_i\dot{\zeta}(t) + \zeta^T(t)\left(\sum_{j=1}^{N}\pi_{ij}\mathscr{P}_j\right)\zeta(t) \\ &\quad + x^T(t)Q_1 x(t) \\ &\quad + x^T(t-d_1)(-Q_1 + Q_2 + Q_3)x(t-d_1) \\ &\quad - \left(1 - \dot{d}(t)\right)x^T(t-d(t))Q_3 x(t-d(t)) \\ &\quad - x^T(t-d_2)Q_2 x(t-d_2) + d_1^2\dot{x}^T(t)R_1\dot{x}(t) \\ &\quad + d_{12}^2\dot{x}^T(t)R_2\dot{x}(t) - d_1\int_{t-d_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds \\ &\quad - d_{12}\int_{t-d_2}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds, \end{aligned} \tag{17}$$

where $\zeta^T(t) = \xi^T(t)\Gamma_1$ and $\dot{\zeta}(t) = \Gamma_2\xi(t)$.

Using the Newton-Leibniz formula, for any N_l and G_l $(l = 1, 2, 3)$ with appropriate dimensions, the following are true:

$$\begin{aligned} 0 &= 2\xi^T(t)\mathscr{N}\left[x(t-d_1) - x(t-d(t)) - \int_{t-d(t)}^{t-d_1}\dot{x}(s)\,ds\right], \\ 0 &= 2\xi^T(t)\mathscr{G}\left[x(t-d(t)) - x(t-d_2) - \int_{t-d_2}^{t-d(t)}\dot{x}(s)\,ds\right], \end{aligned} \tag{18}$$

where $\mathscr{N} = [0\ N_1^T\ N_2^T\ N_3^T\ 0\ 0\ 0]^T$, $\mathscr{G} = [0\ G_1^T\ G_2^T\ G_3^T\ 0\ 0\ 0]^T$.

It can be shown readily that there exists a matrix $M > 0$ such that

$$\begin{aligned} 0 &\leqslant 2\xi^T(t)\mathscr{N}[x(t-d_1) - x(t-d(t))] \\ &\quad + 2\xi^T(t)\mathscr{G}[x(t-d(t)) - x(t-d_2)] \\ &\quad + \frac{d(t) - d_1}{d_{12}}\xi^T(t)\mathscr{N}M^{-1}\mathscr{N}^T\xi(t) \\ &\quad + \frac{d_2 - d(t)}{d_{12}}\xi^T(t)\mathscr{G}M^{-1}\mathscr{G}^T\xi(t) \\ &\quad + d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)M\dot{x}(s)\,ds \\ &\quad + d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)M\dot{x}(s)\,ds. \end{aligned} \tag{19}$$

Adding equalities (18) to the right hand of (17) and considering conditions (2) and (19), we have

$$\begin{aligned}\mathscr{L}V(x_t,i) \leqslant\ & 2\xi^T(t)\Gamma_1\mathscr{P}_i\Gamma_2\xi(t) \\ & + \xi^T(t)\Gamma_1\left(\sum_{j=1}^{N}\pi_{ij}\mathscr{P}_j\right)\Gamma_1^T\xi(t) + x^T(t)Q_1x(t) \\ & + x^T(t-d_1)(-Q_1+Q_2+Q_3)x(t-d_1) \\ & - (1-\mu)x^T(t-d(t))Q_3x(t-d(t)) \\ & - x^T(t-d_2)Q_2x(t-d_2) + d_1^2\dot{x}^T(t)R_1\dot{x}(t) \\ & + d_{12}^2\dot{x}^T(t)R_2\dot{x}(t) \\ & + 2\xi^T(t)\mathscr{N}[x(t-d_1)-x(t-d(t))] \\ & + 2\xi^T(t)\mathscr{G}[x(t-d(t))-x(t-d_2)] \\ & + \frac{d(t)-d_1}{d_{12}}\xi^T(t)\mathscr{N}M^{-1}\mathscr{N}^T\xi(t) \\ & + \frac{d_2-d(t)}{d_{12}}\xi^T(t)\mathscr{G}M^{-1}\mathscr{G}^T\xi(t) \\ & - d_1\int_{t-d_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds \\ & - d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)(R_2-M)\dot{x}(s)\,ds \\ & - d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)(R_2-M)\dot{x}(s)\,ds.\end{aligned} \tag{20}$$

Now, we deal with the integral terms in inequality (20) by applying Lemma 3. Consider

$$-d_1\int_{t-d_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds \leqslant -\begin{bmatrix}\nu_1(t)\\ \nu_2(t)\end{bmatrix}^T\begin{bmatrix}R_1 & 0\\ 0 & 3R_1\end{bmatrix}\begin{bmatrix}\nu_1(t)\\ \nu_2(t)\end{bmatrix}, \tag{21}$$

where $\nu_1(t) = x(t) - x(t-d_1)$, $\nu_2(t) = x(t) + x(t-d_1) - (2/d_1)\int_{t-d_1}^{t} x(s)ds$;

$$\begin{aligned}& - d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds - d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_2\dot{x}(s)\,ds \\ & \leqslant -\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\end{bmatrix}^T\begin{bmatrix}\frac{d_{12}}{d(t)-d_1}\Sigma & 0\\ * & \frac{d_{12}}{d_2-d(t)}\Sigma\end{bmatrix}\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\end{bmatrix},\end{aligned} \tag{22}$$

where $\Sigma = \operatorname{diag}(R_2 - M, 3(R_2 - M))$,

$$\varsigma_1(t) = \begin{bmatrix}x(t-d_1)-x(t-d(t))\\ x(t-d_1)+x(t-d(t))-\frac{2}{d(t)-d_1}\int_{t-d(t)}^{t-d_1}x(s)\,ds\end{bmatrix},$$

$$\varsigma_2(t) = \begin{bmatrix}x(t-d(t))-x(t-d_2)\\ x(t-d(t))+x(t-d_2)-\frac{2}{d_2-d(t)}\int_{t-d_2}^{t-d(t)}x(s)\,ds\end{bmatrix}. \tag{23}$$

Applying Lemma 2 to (22), it yields that if there exists a matrix $X = \begin{bmatrix}X_{11} & X_{12}\\ X_{21} & X_{22}\end{bmatrix}$ with appropriate dimensions such that (9) holds, then

$$\begin{aligned}& - d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds - d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_2\dot{x}(s)\,ds \\ & \leqslant -\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\end{bmatrix}^T\begin{bmatrix}\Sigma & X\\ * & \Sigma\end{bmatrix}\begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\end{bmatrix}.\end{aligned} \tag{24}$$

Combining (20), (21), and (24), we have

$$\begin{aligned}\mathscr{L}V(x_t,i) \leqslant\ & \frac{d(t)-d_1}{d_{12}}\xi^T(t) \\ & \times\left(\Upsilon_1 + d_1^2\Upsilon_3^TR_1\Upsilon_3 + d_{12}^2\Upsilon_3^TR_2\Upsilon_3 + \mathscr{N}M^{-1}\mathscr{N}^T\right) \\ & \times\xi(t) + \frac{d_2-d(t)}{d_{12}}\xi^T(t) \\ & \times\left(\Upsilon_2 + d_1^2\Upsilon_3^TR_1\Upsilon_3 + d_{12}^2\Upsilon_3^TR_2\Upsilon_3 + \mathscr{G}M^{-1}\mathscr{G}^T\right) \\ & \times\xi(t),\end{aligned} \tag{25}$$

where

$$\begin{gathered}\Upsilon_1 = \begin{bmatrix}[\Omega_1]_{5n\times 5n} & [\Omega_2]_{5n\times n} & [\Omega_3]_{5n\times n}\\ * & -12(R_2-M) & -4X_{22}\\ * & * & -12(R_2-M)\end{bmatrix}, \\ \Upsilon_2 = \begin{bmatrix}[\Omega_1]_{5n\times 5n} & [\Omega_2]_{5n\times n} & [\widehat{\Omega}_3]_{5n\times n}\\ * & -12(R_2-M) & -4X_{22}\\ * & * & -12(R_2-M)\end{bmatrix}, \\ \Upsilon_3 = (A_i, A_{di}, 0, 0, 0, 0, 0).\end{gathered} \tag{26}$$

Using the Schur complement formula to (11) and (12), respectively, we have

$$\begin{gathered}\Upsilon_1 + d_1^2\Upsilon_3^TR_1\Upsilon_3 + d_{12}^2\Upsilon_3^TR_2\Upsilon_3 + \mathscr{N}M^{-1}\mathscr{N}^T < 0, \\ \Upsilon_2 + d_1^2\Upsilon_3^TR_1\Upsilon_3 + d_{12}^2\Upsilon_3^TR_2\Upsilon_3 + \mathscr{G}M^{-1}\mathscr{G}^T < 0.\end{gathered} \tag{27}$$

By using convex combination approach [15], inequalities (27) imply that $\mathscr{L}V(x_t,i) < 0$, which implies there exists a sufficiently small $\epsilon > 0$ such that $\mathscr{L}V(x_t,i) \leqslant -\epsilon\|x(t)\|^2$ for any initial condition (x_0, r_0). Therefore, for any $t > 0$, by Dynkin's formula, we have

$$\mathbb{E}\{V(x_t,i)\} - \{V(x_0,r_0)\} \leqslant -\epsilon\mathbb{E}\left\{\int_0^t \|x(s)\|^2 ds \mid (x_0,r_0)\right\}, \tag{28}$$

which yields

$$\mathbb{E}\left\{\int_0^t \|x(s)\|^2 ds \mid (x_0,r_0)\right\} \leqslant \frac{1}{\epsilon}\{V(x_0,r_0)\} < \infty. \tag{29}$$

The previous inequality means that $\lim_{t\to+\infty}\mathbb{E}\{\|x(t)\|^2 \mid (x_0, r_0)\} = 0$. Thus, system (1) is stochastically stable by Definition 1. □

Remark 5. In order to guarantee the Lyapunov-Krasovskii functional $V(x_t, r_t) > 0$, most authors require the Lyapunov matrix $P_i > 0$ in V_1 (see, e.g., [4–8]). The Lyapunov-Krasovskii functional employed in this paper is very simple, but a less conservative result is developed by using the LMI (10) instead of inequality $\mathscr{P}_i > 0$, which can be seen in Section 4. However, it should be pointed out that the provided result cannot be used when the lower delay bound is considered to be zero because of the existence of $1/d_1$ in LMI (10).

Remark 6. It is well known that the convex combination approach is effective in reducing conservatism in stability analysis. In some literature, the integral terms $-2\xi^T(t)\mathscr{N}\int_{t-d(t)}^{t-d_1}\dot{x}(s)\mathrm{d}s$ and $-2\xi^T(t)\mathscr{G}\int_{t-d(t)}^{t-d_1}\dot{x}(s)\mathrm{d}s$ are estimated by $((d(t)-d_1)/d_{12})\xi^T(t)\mathscr{N}R_2^{-1}\mathscr{N}^T\xi(t) + d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ and $((d_2-d(t))/d_{12})\xi^T(t)\mathscr{G}R_2^{-1}\mathscr{G}^T\xi(t) + d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$, respectively, which make the term $d_{12}\int_{t-d_2}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ in (17) disappear. In order to obtain a less conservative result, we applied Lemma 3 to deal with the integral term. In this case, integral term $d_{12}\int_{t-d_2}^{t-d_1}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ must be reserved. Based on the above consideration, another matrix $M > 0$ is introduced when we estimate the integral terms $-2\xi^T(t)\mathscr{N}\int_{t-d(t)}^{t-d_1}\dot{x}(s)\mathrm{d}s$ and $-2\xi^T(t)\mathscr{G}\int_{t-d(t)}^{t-d_1}\dot{x}(s)\mathrm{d}s$.

When $S = \{1\}$, the Markovian jump system (1) reduces to the following linear system with interval time-varying delay:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d(t)),\\ x(t) &= \phi(t), \quad t\in[-d_2, 0].\end{aligned} \tag{30}$$

Based on the above method, we are now ready to give an improved asymptotic stability criterion for system (30).

Corollary 7. *Given scalars $0 < d_1 < d_2$ and μ, the time-varying delay system (30) is asymptotically stable, if there exist matrix $P_{\iota\kappa} = P_{\kappa\iota}^T \in \mathbb{R}^{n\times n}$, $(\iota,\kappa = 1,2,3)$; symmetric positive definite matrices $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$, $Q_1 \in \mathbb{R}^{n\times n}$, $Q_2 \in \mathbb{R}^{n\times n}$, $Q_3 \in \mathbb{R}^{n\times n}$, and $M \in \mathbb{R}^{n\times n}$; and any matrices $X \in \mathbb{R}^{n\times n}$, $N_l \in \mathbb{R}^{n\times n}$, and $G_l \in \mathbb{R}^{n\times n}$ $(l = 1,2)$ such that*

$$\Psi_1 = \begin{bmatrix} R_2 - M & 0 & X_{11} & X_{12}\\ * & 3(R_2-M) & X_{21} & X_{22}\\ * & * & R_2-M & 0\\ * & * & 0 & 3(R_2-M)\end{bmatrix} > 0,$$

$$\Psi_2 = \begin{bmatrix} P_{11} + 2d_1R_1 + \dfrac{2d_{12}^2}{d_1+d_2}R_2 & P_{12} - 2R_1 & P_{13} - \dfrac{2d_{12}}{d_1+d_2}R_2\\ * & P_{22} + \dfrac{1}{d_1}(Q_1+2R_1) & P_{23}\\ * & * & P_{33} + \dfrac{1}{d_{12}}Q_2 + \dfrac{2}{d_1+d_2}R_2\end{bmatrix} > 0,$$

$$\Psi_3 = \begin{bmatrix} [\Phi_1]_{5n\times 5n} & [\Phi_2]_{5n\times n} & [\Phi_3]_{5n\times n} & [\Phi_4]_{5n\times n} & [\Phi_5]_{5n\times n} & [\Phi_6]_{5n\times n}\\ * & -12(R_2-M) & -4X_{22} & 0 & 0 & 0\\ * & * & -12(R_2-M) & 0 & 0 & 0\\ * & * & * & -R_1 & 0 & 0\\ * & * & * & * & -R_2 & 0\\ * & * & * & * & * & -M\end{bmatrix} < 0, \tag{31}$$

$$\Psi_4 = \begin{bmatrix} [\Phi_1]_{5n\times 5n} & [\widehat{\Phi}_2]_{5n\times n} & [\widehat{\Phi}_3]_{5n\times n} & [\Phi_4]_{5n\times n} & [\Phi_5]_{5n\times n} & [\widehat{\Phi}_6]_{5n\times n}\\ * & -12(R_2-M) & -4X_{22} & 0 & 0 & 0\\ * & * & -12(R_2-M) & 0 & 0 & 0\\ * & * & * & -R_1 & 0 & 0\\ * & * & * & * & -R_2 & 0\\ * & * & * & * & * & -M\end{bmatrix} < 0,$$

where

$$\Phi_1(1,1) = \mathrm{Sym}(P_{11}A + P_{12}) + Q_1 - 4R_1,$$
$$\Phi_1(1,2) = P_{11}A_d,$$
$$\Phi_1(1,s) = \Omega_1(1,s), \quad (s = 3,4),$$
$$\Phi_1(1,5) = 6R_1 + d_1(A^TP_{12} + P_{22}^T),$$
$$\Phi_1(2,s) = \Omega_1(2,s), \quad (s = 1,2,3,4)$$
$$\Phi_1(2,5) = d_1A_d^TP_{12}$$
$$\Phi_1(3,s) = \Omega_1(3,s), \quad (s = 3,4,5),$$

$$\Phi_1(4,s) = \Omega_1(4,s), \quad (s = 4,5),$$
$$\Phi_1(5,5) = \Omega_1(5,5),$$
$$\Phi_2(1,1) = d_{12}\left(A^T P_{13} + P_{23}\right),$$
$$\Phi_2(2,1) = d_{12}A_d^T P_{13} + 2X_{21}^T + 2X_{22}^T + 6R_6 - 6M,$$
$$\Phi_2(s,1) = \Omega_2(s,1), \quad (s = 3,4,5),$$
$$\Phi_3(s,1) = \Omega_3(s,1), \quad (s = 1,2,3,4,5),$$
$$\Phi_4(1,1) = d_1 A^T R_1, \qquad \Phi_4(2,1) = d_1 A_d^T R_1,$$
$$\Phi_4(s,1) = \Omega_4(s,1), \quad (s = 3,4,5),$$
$$\Phi_5(1,1) = d_{12} A^T R_2, \qquad \Phi_5(2,1) = d_{12} A_d^T R_2,$$
$$\Phi_5(s,1) = \Omega_5(s,1), \quad (s = 3,4,5),$$
$$\widehat{\Phi}_2(s,1) = \widehat{\Omega}_2(s,1), \quad (s = 1,2,3,4,5),$$
$$\widehat{\Phi}_3(1,1) = d_{12}\left(A^T P_{13} + P_{23}\right),$$
$$\widehat{\Phi}_3(2,1) = d_{12}A_d^T P_{13} + 6R_2 - 6M - 2X_{12} + 2X_{22},$$
$$\widehat{\Phi}_3(s,1) = \widehat{\Omega}_3(s,1), \quad (s = 3,4,5),$$
$$\widehat{\Phi}_6(s,1) = \widehat{\Omega}_6(s,1), \quad (s = 1,2,3,4,5). \tag{32}$$

Remark 8. Theorem 4 and Corollary 7 can be applied to both slow and fast time-varying delays. But when μ is unknown, the above results cannot be used directly to check the stability. From the construction of Lyapunov-Krasovskii functional, it can be seen by setting $Q_3 = 0$ in Theorem 4 and Corollary 7 that the corresponding conclusions are valid for the case when μ is unknown.

4. Numerical Examples

In this section, three numerical examples will be presented to show the validity of the main results derived above.

Example 1. Consider the Markovian jump systems (1) with

$$A_1 = \begin{bmatrix} -2.3 & 0.8 \\ 1 & -2.9 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.8 & 1.2 \\ 0.7 & -3.5 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.9 & 0.2 \\ 0.6 & -0.8 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 1.3 & -2.6 \\ 0.5 & -1.4 \end{bmatrix}. \tag{33}$$

Consider $\pi_{11} = -\pi_{12} = -0.5$, $\pi_{22} = -\pi_{21} = -3$. According to [20], for $d_1 = 0.2$, one obtains $d_2 = 0.520$, while by Theorem 4 in this paper we can obtain $d_2 = 0.605$ such that the system is stochastically stable. More comparisons are shown in Table 1, which indicate that Theorem 4 is much less conservative than that in [20].

Table 1: Admissible upper bounds d_2 with varying d_1.

d_1	0.2	0.25	0.3	0.35	0.4
[20]	0.520	0.537	0.557	0.579	0.603
Theorem 4	0.605	0.615	0.629	0.646	0.667

Example 2. Consider the Markovian jump systems (1) with

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}. \tag{34}$$

This example has been taken from [4]. To compare the stochastic stability condition in Theorem 4 with that in [25, 26], we choose $\pi_{22} = 0.8$, $\mu = 0.9$. Using Theorem 4 of our paper, the admissible upper bound d_2 for different d_1 and π_{11} can be found in Table 2. It can be seen from Table 2 that Theorem 4 in our paper is less conservative.

Example 3. Consider the systems (30) with the following parameters:

$$A = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -0.1 \end{bmatrix}. \tag{35}$$

This system is a well-known delay-dependent stable system where the maximum allowable delay $d_{\max} = 6.1725$ [24]. For known and unknown μ, the admissible upper bounds d_2 for different d_1, which guarantee the asymptotical stability of the system (30), are listed in Tables 3 and 4, respectively. It can be seen from Tables 3 and 4 that the stability results obtained in this paper are less conservative than those in [14, 16, 17, 22, 27, 28].

5. Conclusion

In this paper, the problem of stochastic stability for a class of Markovian jump systems has been investigated. By using the convex combination technique and the improved integral inequality, some less conservative delay-dependent stability criteria are established in terms of linear matrix inequalities.

Table 2: Admissible upper bounds d_2 with varying π_{11} and d_1.

d_1	Methods	$\pi_{11} = -0.1$	$\pi_{11} = -0.3$	$\pi_{11} = -0.5$	$\pi_{11} = -0.7$	$\pi_{11} = -0.9$
0.1	[25]	0.4336	0.4332	0.4328	0.4324	0.4321
	[26]	0.5752	0.5507	0.5375	0.5289	0.5227
	Theorem 4	0.5847	0.5895	0.5949	0.6005	0.6061
0.2	[25]	0.4459	0.4450	0.4442	0.4434	0.4428
	[26]	0.5930	0.5672	0.5529	0.5433	0.5363
	Theorem 4	0.5932	0.5988	0.6048	0.6108	0.6168

Table 3: Admissible upper bounds d_2 with varying μ and d_1.

μ	Methods	$d_1 = 2$	$d_1 = 3$	$d_1 = 4$	$d_1 = 5$	$d_1 = 6$
0.3	[14]	2.6972	3.2591	4.0774	—	—
	[27]	3.0129	3.3408	4.1690	5.0275	—
	[16]	3.1046	3.4181	4.2102	5.0440	—
	Corollary 7	3.2321	3.4977	4.2939	5.1372	6.0071
0.5	[14]	2.5048	3.2591	4.0774	—	—
	[27]	2.5663	3.3408	4.1690	5.0275	—
	[16]	2.6940	3.4181	4.2102	5.0440	—
	Corollary 7	2.800	3.4977	4.2939	5.1372	6.0071
0.9	[14]	2.5048	3.2591	4.0774	—	—
	[27]	2.5663	3.3408	4.1690	5.0275	—
	[16]	2.6940	3.4181	4.2102	5.0440	—
	Corollary 7	2.800	3.4977	4.2939	5.1372	6.0071

Table 4: Admissible upper bounds d_2 with unknown μ and d_1.

d_1	1.0	2.0	3.0	4.0	5.0	6.0
[14]	1.87	2.50	3.25	4.07	—	—
[17]	2.04	2.60	3.30	4.08	—	—
[22]	2.06	2.61	3.31	4.09	—	—
[28]	2.12	2.72	3.45	4.25	5.09	—
Corollary 7 with Remark 8	2.338	2.800	3.497	4.293	5.137	6.007

The numerical examples demonstrate the effectiveness and superiority of the presented results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the Doctoral Research Foundation of Southwest University of Science and Technology (Grant no. 13zx7141).

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