

## Research Article

# Robust Stability Criteria of Roesser-Type Discrete-Time Two-Dimensional Systems with Parameter Uncertainties

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This paper is concerned with robust stability analysis of uncertain Roesser-type discrete-time two-dimensional (2D) systems. In particular, the underlying parameter uncertainties of system parameter matrices are assumed to belong to a convex bounded uncertain domain, which usually is named as the so-called polytopic uncertainty and appears typically in most practical systems. Robust stability criteria are proposed for verifying the robust asymptotical stability of the related uncertain Roesser-type discrete-time 2D systems in terms of linear matrix inequalities. Indeed, a parameter-dependent Lyapunov function is applied in the proof of our main result and thus the obtained robust stability criteria are less conservative than the existing ones. Finally, the effectiveness and applicability of the proposed approach are demonstrated by means of some numerical experiments.

## 1. Introduction

During the past several decades, the well-known Lyapunov stability theory has become an efficient tool for dealing with the problem of stability analysis of many kinds of uncertain systems [1–6]. However, those earlier results on stability analysis of uncertain systems are developed by using the so-called common quadratic Lyapunov function (CQLF) [7]. Actually, the CQLF applies a single Lyapunov matrix for all the submodels and therefore the obtained stability criteria are rather conservative. With the purpose of further releasing the conservatism of the stability criteria, the affine parameter-dependent Lyapunov function (APDLF) has been proposed in [8], where the fixed quadratic Lyapunov function is replaced by a Lyapunov function with affine dependence on the underlying uncertain parameters. Because of the construction of such parameter-dependent Lyapunov functions, the conservatism could be released a lot as a tradeoff.

On the other hand, the famous 2D systems model could represent a wide range of practical plants, for example, water stream heating, thermal processes, biomedical imaging, gas absorption, river pollution modeling, data processing and transmission, process of gas filtration, grid based wireless sensor networks, and so forth, [9, 10]. As a result, a considerable interest in stability analysis of 2D systems has emerged

during the past two decades [11–15]. Recently, the 2D system theory has also been applied to address the problem of stability analysis 2D state-space digital filters with saturation arithmetic in [16–30]. However, it is worth noting that most of the aforementioned results are feasible for linear 2D systems without uncertainties. As is well known, most of the practical 2D dynamical systems in the realistic world are subject to parameter uncertainties and the above results would fail to work when some uncertain parameters occur in the practical settings.

In particular, it is worth noting that the Roesser-type discrete-time 2D system's information is propagated along two independent directions and this fact makes the problem of stability analysis more complicated. Due to the complexity of mathematical analysis of Roesser-type discrete-time 2D systems with parameter uncertainties, there has been little literature which focuses on robust stability analysis of uncertain Roesser-type discrete-time 2D systems so far. Thus, this problem needs to be further investigated and this fact motivates us to carry out this task in this paper.

Based on the above analysis, the problem of robust stability analysis of Roesser-type discrete-time 2D systems with parameter uncertainties will be addressed via the Lyapunov stability theory. The parameter uncertainties of 2D system's

parameter matrices are assumed to belong to a convex bounded uncertain domain, which usually is named as the so-called polytopic uncertainty and appears typically in most modeling processes of uncertainties. An efficient parameter-dependent Lyapunov function is applied in the derivation of our main result and thus the obtained robust stability criteria are less conservative than the existing ones. Moreover, robust stability criteria are given to verify the robust asymptotical stability of the uncertain Roesser-type discrete-time 2D systems in terms of linear matrix inequalities. Finally, the effectiveness and applicability of the proposed approach are demonstrated by means of numerical examples.

The rest of this paper is organized as follows: following the introduction, some preliminaries are provided in Section 2. In Section 3, LMI-based robust stability criteria are proposed for verifying the robust asymptotical stability of the uncertain Roesser-type discrete-time 2D systems. A numerical example is given to demonstrate the effectiveness of the given approach in Section 4. Finally, some conclusions are also given in Section 5.

The following notations are applied for simplicity. A star  $*$  in a symmetric matrix denotes the transposed element in the symmetric position; the symbol  $I$  represents the identity matrix with appropriate dimension;  $X > 0$  (or  $X \geq 0$ ) means the matrix  $X$  is symmetric and positive definite (or symmetric and positive semidefinite);  $X^T$  denotes the transpose of  $X$ .

## 2. Preliminaries

Consider a class of uncertain discrete-time 2D systems which is described by the Roesser-type model

$$\mathbf{x}^+(k, l) = A(\alpha) \mathbf{x}(k, l), \quad (1)$$

with

$$\mathbf{x}(k, l) = \begin{bmatrix} \mathbf{x}^h(k, l) \\ \mathbf{x}^v(k, l) \end{bmatrix}, \quad \mathbf{x}^+(k, l) = \begin{bmatrix} \mathbf{x}^h(k+1, l) \\ \mathbf{x}^v(k, l+1) \end{bmatrix}, \quad (2)$$

where  $k$  and  $l$  are two integers in  $\mathbb{Z}^+$ .  $\mathbf{x}^h(\cdot, \cdot)$  is the horizontal state in  $\mathbf{R}^{n_1}$  and  $\mathbf{x}^v(\cdot, \cdot)$  is the vertical state in  $\mathbf{R}^{n_2}$ , where  $n_1$  and  $n_2$  are dimensions of the horizontal state vector and the vertical state vector, respectively. The system coefficient matrix  $A(\alpha)$  is not precisely known but belongs to a convex bounded uncertain domain:

$$A(\alpha) = \begin{bmatrix} A^{11}(\alpha) & A^{12}(\alpha) \\ A^{21}(\alpha) & A^{22}(\alpha) \end{bmatrix}, \quad (3)$$

with  $A^{11}(\alpha) \in \mathbf{R}^{n_1 \times n_1}$ ,  $A^{12}(\alpha) \in \mathbf{R}^{n_1 \times n_2}$ ,  $A^{21}(\alpha) \in \mathbf{R}^{n_2 \times n_1}$ , and  $A^{22}(\alpha) \in \mathbf{R}^{n_2 \times n_2}$ , respectively. Specially, these matrices  $A^{11}(\alpha)$ ,  $A^{12}(\alpha)$ ,  $A^{21}(\alpha)$ , and  $A^{22}(\alpha)$  belong to a convex bounded (polytopic type) uncertain domain  $\mathcal{P}$  given as follows:

$$\mathcal{P} := \left\{ (A^{11}, A^{12}, A^{21}, A^{22})(\alpha) : (A^{11}, A^{12}, A^{21}, A^{22})(\alpha) = \sum_{i=1}^r \alpha_i (A_i^{11}, A_i^{12}, A_i^{21}, A_i^{22}); \alpha \in \Delta_r \right\}, \quad (4)$$

where  $\Delta_r$  is the so-called unit simplex given by

$$\Delta_r = \left\{ \alpha \in \mathbf{R}^r : \sum_{i=1}^r \alpha_i = 1, \alpha_i \geq 0; i = 1, \dots, r \right\}. \quad (5)$$

Moreover, the boundary conditions along two independent directions are defined as  $\mathbf{x}^h(0, l) = f(l)$  and  $\mathbf{x}^v(k, 0) = g(k)$ , where  $f(l)$  and  $g(k)$  are boundary conditions along the horizontal direction and vertical direction, respectively.

Finally, let us end this section by giving a definition and a lemma which will play an important role in the following proof.

Denote  $X_N = \sup\{\|\mathbf{x}(k, l)\| : N = k + l\}$ , and then we give the definition of robust asymptotical stability for uncertain Roesser-type discrete-time 2D system (1).

*Definition 1.* The uncertain Roesser-type discrete-time 2D system (1) is robust asymptotically stable if  $\lim_{k \rightarrow \infty, l \rightarrow \infty} X_N = 0$  with the initial and boundary conditions  $\mathbf{x}^h(0, l) = f(l)$  and  $\mathbf{x}^v(k, 0) = g(k)$ .

**Lemma 2** (see [7]). *Given matrices  $Q = Q^T$  and  $R = R^T$  with appropriate dimensions, the inequality  $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} > 0$  is equivalent to  $R > 0$ ,  $Q - SR^{-1}S^T > 0$ .*

## 3. Main Results

In this section, by using the Lyapunov stability theory, sufficient robust stability criteria for ensuring the robust asymptotical stability of the underlying uncertain Roesser-type discrete-time 2D system (1) will be proposed in terms of linear matrix inequalities. Indeed, less conservative robust stability conditions are given by means of a parameter-dependent Lyapunov function and a slack method for exploiting the algebraic properties of the uncertain Roesser-type discrete-time 2D system (1).

**Theorem 3.** *The uncertain Roesser-type discrete-time 2D system (1) is robust asymptotically stable if there exist appropriately dimensional matrices  $P_{ij}$ ,  $i = 1, 2, \dots, r$ ;  $i \leq j \leq r$ , with*

$$P_{ij} = \begin{bmatrix} P_{ij}^1 & * \\ P_{ij}^3 & P_{ij}^2 \end{bmatrix}, \quad P_{ij}^1 \in \mathbf{R}^{n_1 \times n_1}, \quad (6)$$

$$P_{ij}^2 \in \mathbf{R}^{n_2 \times n_2}, \quad P_{ij}^3 \in \mathbf{R}^{n_2 \times n_1},$$

such that the following LMIs hold:

$$\begin{bmatrix} -P_{ii} & * \\ P_{ii}A_i & -P_{ii} \end{bmatrix} < 0, \quad i = 1, 2, \dots, r;$$

$$\begin{bmatrix} -P_{ii} & * \\ P_{ii}A_j & -P_{ii} \end{bmatrix} + \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_i & -P_{ij} \end{bmatrix} < 0,$$

$$i = 1, 2, \dots, r-1, \quad i < j;$$

$$\begin{aligned} & \begin{bmatrix} -P_{jj} & * \\ P_{jj}A_i & -P_{jj} \end{bmatrix} + \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_j & -P_{ij} \end{bmatrix} < 0, \\ & i = 1, 2, \dots, r-1, \quad i < j; \\ & \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_l & -P_{ij} \end{bmatrix} + \begin{bmatrix} -P_{il} & * \\ P_{il}A_j & -P_{il} \end{bmatrix} + \begin{bmatrix} -P_{jl} & * \\ P_{jl}A_i & -P_{jl} \end{bmatrix} < 0, \\ & i = 1, 2, \dots, r-2, \quad i < j < l \leq r. \end{aligned} \tag{7}$$

*Proof.* Consider the following parameter-dependent Lyapunov function which is suitable for the uncertain Roesser-type discrete-time 2D system (1):

$$V(\mathbf{x}(k, l)) = \mathbf{x}^T(k, l) P_{\alpha\alpha} \mathbf{x}(k, l), \tag{8}$$

where the matrix  $P_{\alpha\alpha}$  is a positive definite matrix and with the following structure:  $P_{\alpha\alpha} = \sum_{i=1}^r \sum_{i \leq j \leq r} \alpha_i \alpha_j \begin{bmatrix} P_{ij}^1 & * \\ P_{ij}^2 & P_{ij}^3 \end{bmatrix}$ ,  $P^1 \in \mathbf{R}^{n_1 \times n_1}$ ,  $P^2 \in \mathbf{R}^{n_2 \times n_2}$ ,  $P^3 \in \mathbf{R}^{n_2 \times n_1}$ .

Then, the variation of the parameter-dependent Lyapunov function  $V(\mathbf{x}(k, l))$  could be described as

$$\Delta V(\mathbf{x}(k, l)) = \mathbf{x}^T(k, l) (A(\alpha)^T P_{\alpha\alpha} A(\alpha) - P_{\alpha\alpha}) \mathbf{x}(k, l). \tag{9}$$

By applying the Lyapunov stability theory, the uncertain Roesser-type discrete-time 2D system (1) is robust asymptotically stable if the following inequality holds:

$$A(\alpha)^T P_{\alpha\alpha} A(\alpha) - P_{\alpha\alpha} < 0. \tag{10}$$

Applying Lemma 2 to (10), it can be concluded that inequality (10) is equivalent to the following inequality:

$$\Phi = \begin{bmatrix} -P_{\alpha\alpha} & * \\ P_{\alpha\alpha} A(\alpha) & -P_{\alpha\alpha} \end{bmatrix} < 0. \tag{11}$$

On the other hand, reordering the expression of  $\Psi$ , one can obtain

$$\begin{aligned} \Phi &= \begin{bmatrix} -P_{\alpha\alpha} & * \\ P_{\alpha\alpha} A(\alpha) & -P_{\alpha\alpha} \end{bmatrix} \\ &= \sum_{i=1}^r \alpha_i^3 \Phi_{iii} + \sum_{i=1}^{r-1} \sum_{i < j} \alpha_i^2 \alpha_j \Phi_{ijj} \\ &\quad + \sum_{i=1}^{r-1} \sum_{i < j} \alpha_i^2 \alpha_j \Gamma_{ijj} + \sum_{i=1}^{r-2} \sum_{i < j < l} \alpha_i \alpha_j \alpha_l \Phi_{ijl}, \end{aligned} \tag{12}$$

where we have

$$\begin{aligned} \Phi_{iii} &= \begin{bmatrix} -P_{ii} & * \\ P_{ii}A_i & -P_{ii} \end{bmatrix}, \\ \Phi_{ijj} &= \begin{bmatrix} -P_{ii} & * \\ P_{ii}A_j & -P_{ii} \end{bmatrix} + \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_i & -P_{ij} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \Phi_{ijj} &= \begin{bmatrix} -P_{jj} & * \\ P_{jj}A_i & -P_{jj} \end{bmatrix} + \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_j & -P_{ij} \end{bmatrix}, \\ \Phi_{ijl} &= \begin{bmatrix} -P_{ij} & * \\ P_{ij}A_l & -P_{ij} \end{bmatrix} + \begin{bmatrix} -P_{il} & * \\ P_{il}A_j & -P_{il} \end{bmatrix} \\ &\quad + \begin{bmatrix} -P_{jl} & * \\ P_{jl}A_i & -P_{jl} \end{bmatrix}. \end{aligned} \tag{13}$$

From (10)–(12), if the LMI-based stability conditions (7) hold, inequality (10) evidently holds, which guarantee the robust asymptotical stability for the uncertain Roesser-type discrete-time 2D system (1). □

This completes the proof. □

*Remark 4.* From (1) and (4), the parameter uncertainties of 2D system parameter matrices are assumed to belong to a convex bounded uncertain domain. Then, LMI-based robust stability criteria are given for ensuring the robust asymptotical stability of the underlying uncertain Roesser-type discrete-time 2D systems in Theorem 3. Indeed, the parameter-dependent Lyapunov function  $V(\mathbf{x}(k, l)) = \mathbf{x}^T(k, l) P_{\alpha\alpha} \mathbf{x}(k, l)$  is applied in the derivation of our main result and thus the obtained robust stability criteria are less conservative than before. Furthermore, the effectiveness and applicability of the proposed results will be demonstrated by means of numerical experiments in the following section.

### 4. Numerical Examples

Consider the uncertain Roesser-type discrete-time two-dimensional systems described as follows:

$$\begin{bmatrix} x^h(k+1, l) \\ x^v(k, l+1) \end{bmatrix} = \sum_{i=1}^2 \alpha_i \left( A_i \begin{bmatrix} x^h(k, l) \\ x^v(k, l) \end{bmatrix} \right), \tag{14}$$

where  $A_1 = \begin{bmatrix} 1+a_1T_1 & (a_1a_2+a_0)T_1 \\ T_2 & 1+a_2T_2 \end{bmatrix}$  and  $A_2 = \begin{bmatrix} 1+a_1T_1 & a_1a_2T_1 \\ T_2 & 1+a_2T_2 \end{bmatrix}$ . And the following parameter values about  $a_0, a_1, a_2, T_1$ , and  $T_2$  are given:  $a_0 = -2, a_1 = -3, T_1 = 0.1$ , and  $T_2 = 0.2$ . Furthermore, the initial and boundary conditions of the above uncertain Roesser-type discrete-time two-dimensional systems are set as  $x^h(0, l) = 6 \cos(l)$  for  $l < 30$  and  $x^v(k, 0) = 4 \sin(k)$  for  $k < 30$  and  $x^h(0, l) = 0$  for  $l \geq 30$  and  $x^v(k, 0) = 0$  for  $k \geq 30$ .

Let  $a_2 = -0.6$ ; the stability criteria given in Theorem 3 are feasible by solving LMIs (7), which guarantee the robust asymptotical stability for the underlying uncertain Roesser-type discrete-time 2D systems. On the other hand, Figures 1 and 2 show the state trajectory of the system state variables  $x^h(k, l)$  and  $x^v(k, l)$  with  $\alpha_1 = 0.4$  and  $\alpha_2 = 0.6$ , respectively. From Figures 1 and 2, it is easy to see that the state trajectories of  $x^h(k, l)$  and  $x^v(k, l)$  are robust asymptotically stable in this case.

Let  $a_2 = -2.9$ ; the stability criteria given in Theorem 3 are feasible by solving LMIs (7), which guarantee the robust asymptotical stability for the underlying uncertain Roesser-type discrete-time 2D systems. On the other hand, Figures 3 and 4 show the state trajectory of the system state variables

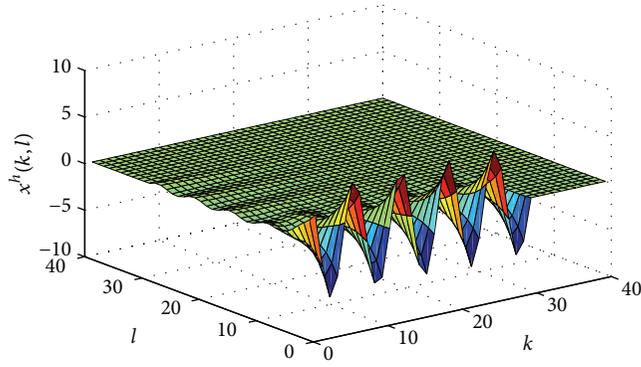


FIGURE 1: The state trajectory of  $x^h(k, l)$  with  $a_2 = -0.6$ .

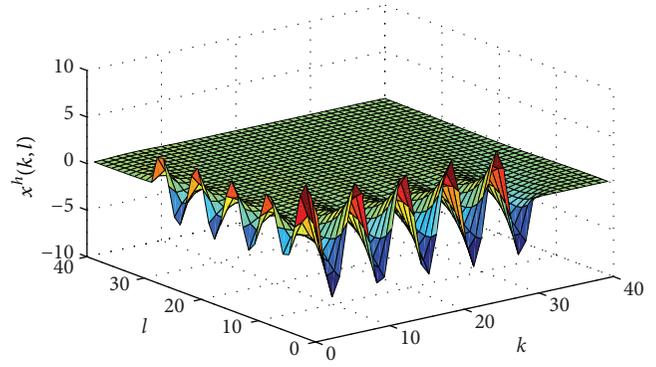


FIGURE 3: The state trajectory of  $x^h(k, l)$  with  $a_2 = -2.9$ .

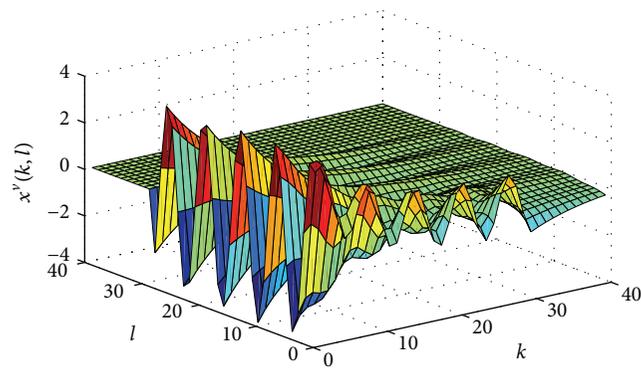


FIGURE 2: The state trajectory of  $x^v(k, l)$  with  $a_2 = -0.6$ .

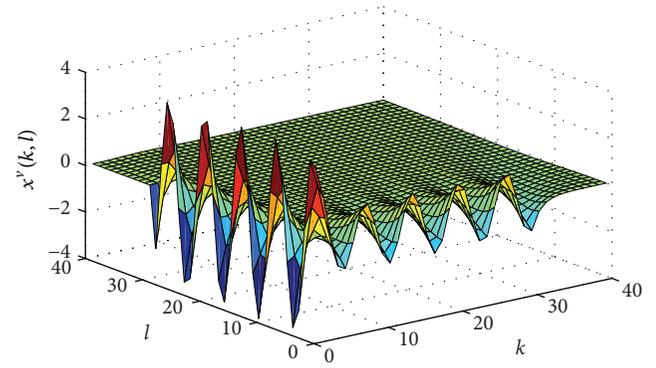


FIGURE 4: The state trajectory of  $x^v(k, l)$  with  $a_2 = -2.9$ .

$x^h(k, l)$  and  $x^v(k, l)$  with  $\alpha_1 = 0.4$  and  $\alpha_2 = 0.6$ , respectively. From Figures 3 and 4, it is easy to see that the state trajectories of  $x^h(k, l)$  and  $x^v(k, l)$  are robust asymptotically stable in this case.

Let  $a_2 = -8.9$ ; the stability criteria given in Theorem 3 are not feasible by solving LMIs (7), which do not guarantee the robust asymptotical stability for the underlying uncertain Roesser-type discrete-time 2D systems. On the other hand, Figures 5 and 6 show the state trajectory of the system state variables  $x^h(k, l)$  and  $x^v(k, l)$  with  $\alpha_1 = 0.4$  and  $\alpha_2 = 0.6$ , respectively. From Figures 5 and 6, it is easy to see that the state trajectories of  $x^h(k, l)$  and  $x^v(k, l)$  are not robust asymptotically stable in this case. Now, it could be concluded that the effectiveness and applicability of the proposed approach given in Theorem 3 are illustrated by means of numerical experiments.

### 5. Conclusions

The problem of robust stability analysis of a class of uncertain Roesser-type discrete-time 2D systems has been addressed by using an efficient parameter-dependent Lyapunov function. In particular, the parameter uncertainties of the underlying 2D system's parameter matrices belong to a convex bounded uncertain domain, which often is named as polytopic uncertainty and appears typically in most practical systems. In order to ensure the robust asymptotic stability of the

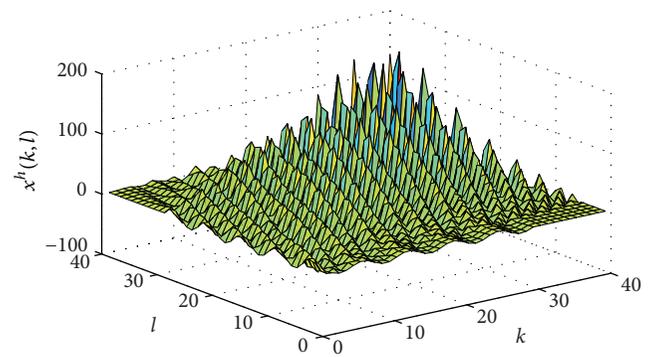


FIGURE 5: The state trajectory of  $x^h(k, l)$  with  $a_2 = -8.9$ .

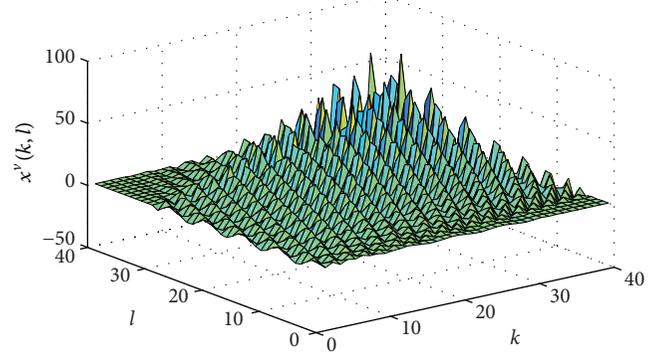


FIGURE 6: The state trajectory of  $x^v(k, l)$  with  $a_2 = -8.9$ .

uncertain Roesser-type discrete-time 2D systems, LMI-based robust stability criteria are proposed by exploiting the algebraic properties of the convex bounded uncertain domain. Finally, a numerical example is provided to demonstrate the effectiveness and applicability of the approach given in this paper.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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