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Research Article

BIBO Stabilization of Discrete-Time Stochastic Control Systems with Mixed Delays and Nonlinear Perturbations

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The problem of bounded-input bounded-output (BIBO) stabilization in mean square for a class of discrete-time stochastic control systems with mixed time-varying delays and nonlinear perturbations is investigated. Some novel delay-dependent stability conditions for the previously mentioned system are established by constructing a novel Lyapunov-Krasovskii function. These conditions are expressed in the forms of linear matrix inequalities (LMIs), whose feasibility can be easily checked by using MATLAB LMI Toolbox. Finally, a numerical example is given to illustrate the validity of the obtained results.

1. Introduction

Many dynamical systems not only depend on the present states but also involve the past ones, generally called the time-delay systems. Generally, as a source of poor or significantly deteriorated performance and instability for the concerned closed-loop system, the time delays are unavoidable in technology and nature. Many works have been done on the stability of time-delay systems; one can see [1–23] and the references therein. The dynamics analysis of continuous-time systems with distributed delay has been well studied in [9–12, 20]. The aspect of simulation and application in control systems, whereas, discrete-time control systems play a more important role than their continuous-time counterparts in the practical digital world. If one wants to simulate or compute the continuous-time systems, it is essential to formulate the discrete-time analogue so as to investigate the dynamical characteristics. It is necessary to take continuous distributed delays into account for modeling realistic systems, for example, neural networks; due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, a neural network usually has a spatial nature. Very recently, Liu et al. introduced the infinite distributed delay and distributed delay in the form of constant delay into the delay neural networks. See [17–19].

In order to track out the reference input signal in real world, the bounded-input bounded-output stabilization has been investigated by many researchers, one can see [20–32] and the references therein. In [22, 23], the sufficient conditions for BIBO stabilization of control systems with no delays were proposed by the Bihari type inequality. In [9, 10], by employing the parameters technique and the Gronwall inequality, the authors investigated the BIBO stability of the systems without distributed time delays. In [20, 27, 29], based on Riccati equations and by constructing appropriate Lyapunov functions, some BIBO stabilization conditions for a class of delayed control systems with nonlinear perturbations were established. In [30], the BIBO stabilization problem of a class of piecewise switched linear systems was further investigated. It should be pointed out that almost all results concerning the BIBO stability for control systems mainly concentrate on continuous-time models. Seldom works have been done for discrete-time control systems one can see [21, 28]. In addition, the previously mentioned works just considered the deterministic systems (see, e.g., [31, 32]). The deterministic systems often fluctuate due to noise, which is random or at least appears to be so. Therefore, we must move from deterministic problems to stochastic ones. So, the BIBO stabilization for stochastic control systems case is necessary and interesting. To the best of our knowledge, there is no work reported on the mean square BIBO stabilization for the discrete-time stochastic control systems with mixed time-varying delays.

It is well known that the classical technique applied in the study of stability is based on the Lyapunov direct method. However, the Lyapunov direct method has some difficulties with the theory and application to specific problems while discussing the stability of solutions in stochastic systems with time delay. In [33], the midpoint in the time delay's variation interval is introduced, and the variation interval is divided into two subintervals with equal length, by constructing the Lyapunov functional which involved midpoint to reduce the conservatism of stability conditions. This method was first proposed to study the stability and stabilization problems for linear continuous-time systems, and then many successful applications were found in [13–15]. In this paper, we will reconsider this method by introducing a new piecewise-like delay method, given that the point of the time delay's variation interval is arbitrary point rather than midpoint.

Motivated by the aforementioned works, in this paper, we investigate BIBO stabilization in mean square for a class of discrete-time stochastic control systems with mixed time-varying delays and nonlinear perturbations. Some novel delay-dependent stability conditions for the previously mentioned system are derived by constructing a novel Lyapunov-Krasovskii function. These conditions are expressed in the forms of linear matrix inequalities, whose feasibility can be easily checked by using MATLAB LMI Toolbox. Finally, a numerical example is given to illustrate the validity of the obtained results.

The paper is organized as follows. In Section 2, some notations and the problem formulation are proposed. The main results are given in Section 3. In Section 4, a numerical example is given to illustrate the validity of the obtained theory results. The conclusion is proposed in Section 5.

2. Notations and Problem Formulation

Firstly, we propose some notations which will be needed in the sequel. The notations are quite standard. Let R^n and $R^{n\times m}$ denote, respectively, the n-dimensioned Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respective $X > Y$) means that X and Y are symmetric matrices and that $X-Y$ is positive semidefinitive (respective positive definite). Let $\|\cdot\|$ denote the Euclidean norm in R^n, let N^+ denote the positive integer set, and let I be the identity matrix with compatible dimension. If A is a matrix, denote by $\|A\|$ its operator norm; that is, $\|A\| = \sup\{\|Ax\| : \|x\| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$, where $\lambda_{\max}(A)$ (resp., $\lambda_{\min}(A)$) means the largest (resp., smallest) value of A. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. The asterisk $*$ in a matrix is used to denote term that is induced by its symmetry. Matrices, if not explicitly state are assumed to have compatible dimensions. Denote $N[a,b] := \{a, a+1, \dots, b\}$. Sometimes, the arguments of the functions will be omitted in the analysis without confusions.

In this paper, we consider the discrete-time stochastic control system with mixed time-varying delays and nonlinear perturbations with the following form:

$$\begin{aligned} x(k+1) = {} & Ax(k) + Bx(k-\tau(k)) + Cu(k) \\ & + D\sum_{i=-d(k)}^{-1} h(x(k+i)) + f(k, x(k), x(k-\tau(k))) \\ & + (Gx(k) + Hx(k-d(k)))\,\omega(k), \\ y(k) = {} & Mx(k), \end{aligned} \tag{1}$$

where $x(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T \in R^n$ denotes the state vector, $u(k) = [u_1(k), u_2(k), \dots, u_m(k)]^T \in R^m$ is the control input vector, $y(k) = [y_1(k), y_2(k), \dots, y_n(k)]^T \in R^n$ is the control output vector, $h(x(k)) = [h_1(x(k)), h_2(x(k)), \dots, h_n(x(k))]^T$, A, B, D, G, C, H, and M represent the weighting matrices with appropriate dimension, and the positive integers $\tau(k)$ and $d(k)$ are the discrete-time-varying delay, distributed time-varying delay and respectively, satisfying that

$$\tau_1 \leq \tau(k) \leq \tau_2, \quad d_1 \leq d(k) \leq d_2, \quad k \in N^+, \tag{2}$$

with τ_1, τ_2, d_1, and d_2 being four known positive integers. For any given $\tau^* \in (\tau_1, \tau_2)$, $d^* \in (d_1, d_2)$. The initial conditions of the system (1) are given by

$$x(k) = \phi(k), \quad k \in [-\max\{\tau_2, d_2\}, 0]. \tag{3}$$

The nonlinear vector-valued perturbation $f(k, x(k), x(k-\tau(k)))$ satisfies that

$$\begin{aligned} & \|f(k, x(k), x(k-\tau(k)))\|^2 \\ & \quad \leq \alpha_1\|x(k)\|^2 + \alpha_2\|x(k-\tau(k))\|^2, \end{aligned} \tag{4}$$

where α_1 and α_2 are two positive constants. $\omega(k)$ is a scalar Wiener process defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with

$$\begin{aligned} & E(\omega(k)) = 0, \qquad E\left(\omega(k)^2\right) = 1, \\ & E(\omega(i)\,\omega(j)) = 0, \quad i \neq j. \end{aligned} \tag{5}$$

Remark 1. The τ^* divides the discrete-time delay's variation interval into two subintervals, that is, $[\tau_1, \tau^*]$ and $(\tau^*, \tau_2]$, and d^* divides the distributed time delay's variation interval into two subintervals, that is, $[d_1, d^*]$ and $(d^*, d_2]$. We will discuss the variation of differences of the Lyapunov-Krasovskii functional $V(t, x(t))$ for each subinterval. Compared with the previous results in the works of [27–32], the BIBO stability conditions are derived in this paper by checking the variation of $V(t, x(t))$ in subintervals rather than in the whole variation interval of the delays.

In what follows, we describe the controller with the form

$$u(k) = Kx(k) + r(k), \tag{6}$$

where K is the feedback gain matrix and $r(k)$ is the reference input.

Assumption 2. For any $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$, let

$$\gamma_i^- \leq \frac{h_i(\xi_1) - h_i(\xi_2)}{\xi_1 - \xi_2} \leq \gamma_i^+, \tag{7}$$

where γ_i^- and γ_i^+ are known constant scalars.

Remark 3. The constants γ_i^-, γ_i^+ in Assumption 2 are allowed to be positive, negative, or zero. Hence, the function $h(x(k))$ could be nonmonotonic and is more general than the usual sigmoid functions and the recently commonly used Lipschitz conditions.

At the end of this section, let us introduce some important definitions and lemmas as which will be used in the sequel.

Definition 4 (see [28, 31]). A vector function $r(k) = (r_1(k), r_2(k), \ldots, r_n(k))^T$ is said to be an element of L_∞^n, if $\|r\|_\infty = \sup_{k\in N[0,\infty)}\|r(k)\| < +\infty$, where $\|\cdot\|$ denotes the Euclid norm in R^n, or the norm of a matrix.

Definition 5 (see [28, 31]). The nonlinear stochastic control system (1) is said to be BIBO stability in mean square, if we can construct a controller (6) such that the output $y(k)$ satisfies that

$$\mathbb{E}\|y(k)\|^2 \leq N_1 + N_2\|r\|_\infty^2, \tag{8}$$

where N_1 and N_2 are two positive constants.

Lemma 6 (see [28]). *For any given vectors $v_i \in R^n$, $i = 1, 2, \ldots, n$, the following inequality holds:*

$$\left[\sum_{i=1}^n v_i\right]^T \left[\sum_{i=1}^n v_i\right] \leq n\sum_{i=1}^n v_i^T v_i. \tag{9}$$

Lemma 7 (see [28]). *Let $x, y \in R^n$ and any $n \times n$ positive-definite matrix $Q > 0$. Then, one has*

$$2x^T y \leq x^T Q^{-1} x + y^T Q y. \tag{10}$$

Lemma 8 (see [28]). *Given the constant matrices Ω_1, Ω_2, and Ω_3 with appropriate dimensions, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ if and only if*

$$\begin{pmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{pmatrix} < 0. \tag{11}$$

3. BIBO Stabilization for the System (1)

In this section, we aim to establish our main results based on the LMI approach. For the conveniences, we denote

$$\begin{aligned}
&\Gamma_1 = \operatorname{diag}\{\gamma_1^-\gamma_1^+, \gamma_2^-\gamma_2^+, \ldots, \gamma_n^-\gamma_n^+\}, \\
&\Gamma_3 = \operatorname{diag}\{\gamma_1^+, \gamma_2^+, \ldots, \gamma_n^+\}, \\
&\Gamma_2 = \operatorname{diag}\left\{\frac{\gamma_1^- + \gamma_1^+}{2}, \frac{\gamma_2^- + \gamma_2^+}{2}, \ldots, \frac{\gamma_n^- + \gamma_n^+}{2}\right\}, \\
&a = \tau_2 - \tau_1 + 1, \\
&b = \begin{cases} \dfrac{(d^* + d_1 - 1)(d^* - d_1) + 2d^*}{2}, & d_1 \leq d(k) \leq d^* \\ \dfrac{(d^* + d_2 - 1)(d_2 - d^*) + 2d_2}{2}, & d^* < d(k) \leq d_2, \end{cases} \\
&c = \begin{cases} d^*, & d_1 \leq d(k) \leq d^* \\ d_2, & d^* < d(k) \leq d_2, \end{cases} \\
&\theta(k) = \begin{cases} x(k - \tau_1), & \tau_1 \leq \tau(k) \leq \tau^* \\ x(k - \tau_2), & \tau^* < \tau(k) \leq \tau_2, \end{cases} \\
&\tilde{\tau} = \begin{cases} \tau^* - \tau_1, & \tau_1 \leq \tau(k) \leq \tau^* \\ \tau_2 - \tau^*, & \tau^* < \tau(k) \leq \tau_2, \end{cases} \\
&\beta = \begin{cases} \dfrac{\tau(k) - \tau_1}{\tau^* - \tau_1}, & \tau_1 \leq \tau(k) \leq \tau^* \\ \dfrac{\tau_2 - \tau(k)}{\tau_2 - \tau^*}, & \tau^* < \tau(k) \leq \tau_2. \end{cases}
\end{aligned} \tag{12}$$

Theorem 9. *For given positive integers τ_1, τ_2, d_1, and d_2, under Assumption 2, the nonlinear discrete-time stochastic control system (1) with the controller (6) is BIBO stabilization in mean square, if there exist symmetric positive-definite matrix P, R, Q_i, $i = 1, 2, \ldots, 5$, Z_1, Z_2, and X with appropriate dimensional, positive-definite diagonal matrices Λ and some positive constants ζ and λ^* such that the following two LMIs hold:*

$$P + 2\left(\tau^{*2}Z_1 + \tilde{\tau}^2 Z_2\right) \leq \lambda^* I, \tag{13}$$

$$\begin{aligned}
\Xi &= \begin{pmatrix}
\Xi_{11} & \Xi_{12} & Z_1 & 0 & 0 & \Gamma_2\Lambda & \Xi_{17} & \Xi_{18} & \sqrt{2}X \\
* & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & B^T D & B^T & 0 \\
* & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Xi_{55} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & \Xi_{77} & \lambda^* D^T & 0 \\
* & * & * & * & * & * & * & -\zeta I & 0 \\
* & * & * & * & * & * & * & * & -\lambda^* I
\end{pmatrix} \\
&\leq 0,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\Xi_{11} &= Q_1 + Q_2 + Q_3 + a\left(Q_3 + Q_4\right) - Z_1 \\
&\quad + Q_5 - \Gamma_1 \Lambda + G^T \lambda^* G + 2\tau^{*2} Z_1 + 2\tilde{\tau}^2 Z_2 \\
&\quad + 2\lambda^* \alpha_1 I + \alpha_1 \zeta I - P + 2\lambda^* A^T A - 2A^T X - 2X^T A, \\
\Xi_{12} &= \lambda^* A^T B - X^T B + G^T \lambda^* H, \\
\Xi_{17} &= \lambda^* A^T D - X^T D, \\
\Xi_{18} &= \lambda^* A^T - X^T, \\
\Xi_{22} &= H^T \lambda^* H + 2\lambda^* B^T B - Q_4 - 2Z_2 - \beta Z_2 \\
&\quad + 2\lambda^* \alpha_2 I + \alpha_2 \zeta I - \left(1 - \beta\right) Z_2, \\
\Xi_{23} &= Z_2 + \beta Z_2, \\
\Xi_{25} &= Z_2 + \left(1 - \beta\right) Z_2, \\
\Xi_{33} &= -Q_1 - Z_1 - Z_2 - \beta Z_2, \\
\Xi_{55} &= -Q_5 - Z_2 - \left(1 - \beta\right) Z_2, \\
\Xi_{66} &= bR - \Lambda, \\
\Xi_{77} &= 2\lambda^* D^T D - \frac{1}{c} R, \\
X &= -\lambda^* C K.
\end{aligned} \tag{15}$$

Proof. We construct the following Lyapunov-Krasovskii function for the system (1):

$$\begin{aligned}
V\left(k, x\left(k\right)\right) &= V_1\left(k, x\left(k\right)\right) + V_2\left(k\right) + V_3\left(k\right) \\
&\quad + V_4\left(k\right) + V_5\left(k\right) + V_6\left(k\right),
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
V_1\left(k, x\left(k\right)\right) &= x^T\left(k\right) P x\left(k\right), \\
V_2\left(k\right) &= \sum_{i=k-\tau^*}^{k-1} x^T\left(i\right) Q_1 x\left(i\right) \\
&\quad + \sum_{i=k-d^*}^{k-1} x^T\left(i\right) Q_2 x\left(i\right), \\
V_3\left(k\right) &= \sum_{i=k-\tau(k)}^{k-1} x^T\left(i\right) Q_3 x\left(i\right) + \sum_{i=\tau_1}^{\tau_2-1} \sum_{j=k-i}^{k-1} x^T\left(j\right) Q_3 x\left(j\right) \\
&\quad + \sum_{i=k-\tau_1}^{k-1} x^T\left(i\right) Q_3 x\left(i\right) \\
&\quad + \sum_{i=-\tau_2+1}^{-\tau_1+1} \sum_{j=k-1+i}^{k-1} x^T\left(j\right) Q_4 x\left(j\right), \\
V_4\left(k\right) &= \tau^* \sum_{i=-\tau^*}^{-1} \sum_{j=k+i}^{k-1} \eta^T\left(j\right) Z_1 \eta\left(j\right), \\
\eta\left(k\right) &= x\left(k+1\right) - x\left(k\right),
\end{aligned}$$

$$V_5\left(k\right) = \begin{cases}
\displaystyle\sum_{i=k-\tau_1}^{k-1} x^T\left(i\right) Q_5 x\left(i\right) + \left(\tau^* - \tau_1\right) \times \sum_{i=-\tau^*}^{-\tau_1-1} \sum_{j=k+i}^{k-1} \eta^T\left(j\right) Z_2 \eta\left(j\right), & \tau_1 \le \tau\left(k\right) \le \tau^* \\
\displaystyle\sum_{i=k-\tau_2}^{k-1} x^T\left(i\right) Q_5 x\left(i\right) + \left(\tau_2 - \tau^*\right) \times \sum_{i=-\tau_2}^{-\tau^*-1} \sum_{j=k+i}^{k-1} \eta^T\left(j\right) Z_2 \eta\left(j\right), & \tau^* < \tau\left(k\right) \le \tau_2,
\end{cases}$$

$$V_6\left(k\right) = \begin{cases}
\displaystyle\sum_{i=-d(k)}^{-1} \sum_{j=k+i}^{k-1} h^T\left(x\left(j\right)\right) R h\left(x\left(j\right)\right) + \sum_{i=-d^*}^{-d_1-1} \sum_{j=i+1}^{-1} \sum_{l=k+j}^{k-1} h^T\left(x\left(l\right)\right) R h\left(x\left(l\right)\right), & d_1 \le d\left(k\right) \le d^* \\
\displaystyle\sum_{i=-d(k)}^{-1} \sum_{j=k+i}^{k-1} h^T\left(x\left(j\right)\right) R h\left(x\left(j\right)\right) + \sum_{i=-d_2}^{-d^*-1} \sum_{j=i+1}^{-1} \sum_{l=k+j}^{k-1} h^T\left(x\left(l\right)\right) R h\left(x\left(l\right)\right), & d^* < d\left(k\right) \le d_2.
\end{cases} \tag{17}$$

Calculating the difference of $V(k, x(k))$ and taking the mathematical expectation, by Lemma 6, we have

$$\begin{aligned}
& E\Delta V_1\left(k, x\left(k\right)\right) \\
&\quad = E\left[x^T\left(k+1\right) P x\left(k+1\right) - x^T\left(k\right) P x\left(k\right)\right] \\
&\quad = E\left[\eta^T\left(k\right) P \eta\left(k\right) + 2\eta^T\left(k\right) P x\left(k\right)\right], \\
& E\Delta V_2\left(k\right) \\
&\quad = E\left[x^T\left(k\right) Q_1 x\left(k\right) - x^T\left(k-\tau^*\right) Q_1 x\left(k-\tau^*\right)\right. \\
&\qquad \left. + x^T\left(k\right) Q_2 x\left(k\right) - x^T\left(k-d^*\right) Q_2 x\left(k-d^*\right)\right], \\
& E\Delta V_3\left(k\right) \\
&\quad = E\left[\left(\sum_{i=k+1-\tau(k+1)}^{k} - \sum_{i=k-\tau(k)}^{k-1}\right) x^T\left(i\right) Q_3 x\left(i\right)\right. \\
&\qquad + \sum_{i=\tau_1}^{\tau_2-1}\left(\sum_{j=k-i+1}^{k} - \sum_{j=k-i}^{k-1}\right) x^T\left(j\right) Q_3 x\left(j\right) \\
&\qquad + \left(\sum_{i=k-\tau_1+1}^{k} - \sum_{i=k-\tau_1}^{k-1}\right) x^T\left(i\right) Q_3 x\left(i\right) \\
&\qquad + \sum_{i=-\tau_2+1}^{-\tau_1+1}\left(\sum_{j=k+i}^{k} - \sum_{j=k+j-1}^{k-1}\right) \\
&\qquad \left. \times x^T\left(j\right) Q_4 x\left(j\right)\right]
\end{aligned}$$

$$\leq E\left[\left(\sum_{i=k+1-\tau_2}^{k} x^T(i)Q_3x(i) - \sum_{i=k-\tau_1}^{k-1} x^T(i)Q_3x(i)\right)\right.$$
$$+\sum_{i=\tau_1}^{\tau_2-1}\left(x^T(k)Q_3x(k) - x^T(k-i)Q_3x(k-i)\right)$$
$$+\left(x^T(k)Q_3x(k) - x^T(k-\tau_1)Q_3x(k-\tau_1)\right)$$
$$+\sum_{i=-\tau_2+1}^{-\tau_1+1}\left(x^T(k)Q_4x(k) - x^T\right.$$
$$\left.\left.\times(k+i-1)Q_4x(k+i-1)\right)\right]$$
$$= E\left[x^T(k)\left[a(Q_3+Q_4)+Q_3\right]x(k)\right.$$
$$-2x^T(k-\tau_1)Q_3x(k-\tau_1)$$
$$\left.-\sum_{i=k-\tau_2}^{k-\tau_1} x^T(i)Q_4x(i)\right]$$
$$\leq E\left[x^T(k)\left[a(Q_3+Q_4)+Q_3\right]x(k)\right.$$
$$\left.-x^T(k-\tau(k))Q_4x(k-\tau(k))\right],$$
$$E\Delta V_4(k)$$
$$= E\left[\tau^*\sum_{i=-\tau^*}^{-1}\left(\sum_{j=k+i+1}^{k} - \sum_{j=k+i}^{k-1}\right)\eta^T(j)Z_1\eta(j)\right]$$
$$= E\left[\tau^{*2}\eta^T(k)Z_1\eta(k) - \tau^*\sum_{i=k-\tau^*}^{k-1}\eta^T(i)Z_1\eta(i)\right]$$
$$\leq E\left[\tau^{*2}\eta^T(k)Z_1\eta(k)\right.$$
$$\left.-\sum_{i=k-\tau^*}^{k-1}\eta^T(i)Z_1\sum_{i=k-\tau^*}^{k-1}\eta(i)\right]. \tag{18}$$

Note that

$$-\sum_{i=k-\tau^*}^{k-1}\eta^T(i)Z_1\sum_{i=k-\tau^*}^{k-1}\eta(i)$$
$$=\begin{pmatrix} x(k) \\ x(k-\tau^*)\end{pmatrix}^T\begin{pmatrix} -Z_1 & Z_1 \\ * & -Z_1\end{pmatrix}\begin{pmatrix} x(k) \\ x(k-\tau^*)\end{pmatrix}, \tag{19}$$

$$E\Delta V_5(k) = E\left\{x^T(k)Q_5x(k) - \theta^T(k)Q_5\theta(k)\right.$$
$$\left.+\tilde{\tau}^2\eta^T(k)Z_2\eta(k) - \tilde{\tau}\psi(k)\right\}, \tag{20}$$

where

$$\psi(k) = \begin{cases}\displaystyle\sum_{i=k-\tau^*}^{k-\tau_1-1}\eta^T(i)Z_2\eta(i), & \tau_1 \leq \tau(k) \leq \tau^*, \\ \displaystyle\sum_{i=k-\tau_2}^{k-\tau^*-1}\eta^T(i)Z_2\eta(i), & \tau^* < \tau(k) \leq \tau_2.\end{cases} \tag{21}$$

When $\tau^* < \tau(k) \leq \tau_2$, it is easy to compute that

$$-\tilde{\tau}\psi(k)$$
$$= -\left[(\tau_2-\tau(k)) + (\tau(k)-\tau^*)\right]$$
$$\times\sum_{i=k-\tau(k)}^{k-\tau^*-1}\eta^T(i)Z_2\eta(i)$$
$$-\left[((\tau_2-\tau(k)) + (\tau(k)-\tau^*))\right]$$
$$\times\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i)Z_2\eta(i)$$
$$\leq -\beta\sum_{i=k-\tau(k)}^{k-\tau^*-1}\eta^T(i)Z_2\sum_{i=k-\tau(k)}^{k-\tau^*-1}\eta(i)$$
$$-\sum_{i=k-\tau(k)}^{k-\tau^*-1}\eta^T(i)Z_2\sum_{i=k-\tau(k)}^{k-\tau^*-1}\eta(i)$$
$$-\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i)Z_2\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta(i) - (1-\beta)$$
$$\times\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta^T(i)Z_2\sum_{i=k-\tau_2}^{k-\tau(k)-1}\eta(i). \tag{22}$$

When $\tau_1 \leq \tau(k) \leq \tau^*$, similarly we can have

$$-\tilde{\tau}\psi(k) \leq -(1-\beta)\sum_{i=k-\tau(k)}^{k-\tau_1-1}\eta^T(i)Z_2\sum_{i=k-\tau(k)}^{k-\tau_1-1}\eta(i)$$
$$-\sum_{i=k-\tau(k)}^{k-\tau_1-1}\eta^T(i)Z_2\sum_{i=k-\tau(k)}^{k-\tau-1}\eta(i)$$
$$-\sum_{i=k-\tau^*}^{k-\tau(k)-1}\eta^T(i)Z_2\sum_{i=k-\tau^*}^{k-\tau(k)-1}\eta(i)$$
$$-\beta\sum_{i=k-\tau^*}^{k-\tau(k)-1}\eta^T(i)Z_2\sum_{i=k-\tau^*}^{k-\tau(k)-1}\eta(i). \tag{23}$$

From (20), (22), and (23), we have

$$\begin{aligned}
E\Delta V_5(k) = E\Bigg[& x^T(k)Q_5x(k) - \theta^T(k)Q_5\theta(k) \\
& + \tilde{\tau}^2\eta^T(k)Z_2\eta(k) + \begin{pmatrix} x(k-\tau(k)) \\ x(k-\tau^*) \\ \theta(k) \end{pmatrix}^T \\
& \times \begin{pmatrix} -2Z_2 & Z_2 & Z_2 \\ * & -Z_2 & 0 \\ * & * & -Z_2 \end{pmatrix} \begin{pmatrix} x(k-\tau(k)) \\ x(k-\tau^*) \\ \theta(k) \end{pmatrix} \\
& + \beta \begin{pmatrix} x(k-\tau(k)) \\ x(k-\tau^*) \end{pmatrix}^T \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \\
& \times \begin{pmatrix} x(k-\tau(k)) \\ x(k-\tau^*) \end{pmatrix} + (1-\beta) \\
& \times \begin{pmatrix} x(k-\tau(k)) \\ \theta(k) \end{pmatrix}^T \begin{pmatrix} -Z_2 & Z_2 \\ * & -Z_2 \end{pmatrix} \\
& \times \begin{pmatrix} x(k-\tau(k)) \\ \theta(k) \end{pmatrix} \Bigg].
\end{aligned} \tag{24}$$

When $d_1 \le d(k) \le d^*$, by Lemma 6, it is easy to get

$$\begin{aligned}
E\Delta V_6(k) = E\Bigg[& \left(\sum_{i=-d(k+1)}^{-1} \sum_{j=k+i+1}^{k} - \sum_{i=-d(k)}^{-1} \sum_{j=k+i}^{k-1} \right) \\
& \times h^T(x(j))Rh(x(j)) \\
& + \sum_{i=-d^*}^{-d_1-1} \sum_{j=i+1}^{-1} \left(\sum_{l=k+j+1}^{k} - \sum_{l=k+j}^{k-1} \right) \\
& \times h^T(x(l))Rh(x(l)) \Bigg] \\
\le E\Bigg[& \sum_{i=-d^*}^{-1} \sum_{j=k+i+1}^{k-1} h^T(x(j))Rh(x(j)) \\
& - \sum_{i=-d(k)}^{-1} \sum_{j=k+i+1}^{k-1} h^T(x(j))Rh(x(j)) \\
& + \sum_{i=-d^*}^{-1} h^T(x(k))Rh(x(k)) \\
& - \sum_{i=-d(k)}^{-1} h^T(x(k+i))Rh(x(k+i)) \\
& + \sum_{i=-d^*}^{-d_1-1} \sum_{j=i+1}^{-1} h^T(x(k))Rh(x(k)) \\
& - \sum_{i=-d^*}^{-d_1-1} \sum_{j=i+1}^{-1} h^T(x(k+j))Rh(x(k+j)) \Bigg] \\
\le E\Bigg[& \frac{(d^*+d_1-1)(d^*-d_1)+2d^*}{2} \\
& \times h^T(x(k))Rh(x(k)) \\
& - \frac{1}{d^*} \left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right)^T \\
& \times R\left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right) \Bigg].
\end{aligned} \tag{25}$$

When $d^* < d(k) \le d_2$, similarly we can have

$$\begin{aligned}
E\Delta V_6(k) \le E\Bigg[& \frac{(d^*+d_2-1)(d_1-d^*)+2d_2}{2} \\
& \times h^T(x(k))Rh(x(k)) \\
& - \frac{1}{d_2} \left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right)^T \\
& \times R\left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right) \Bigg].
\end{aligned} \tag{26}$$

From (25) and (26), we have

$$\begin{aligned}
E\Delta V_6(k) \le E\Bigg[& bh^T(x(k))Rh(x(k)) \\
& - \frac{1}{c} \left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right)^T \\
& \times R\left(\sum_{i=-d(k)}^{-1} h(x(k+i)) \right) \Bigg].
\end{aligned} \tag{27}$$

From (7), it follows that

$$\begin{aligned}
& (h_i(x(k)) - \gamma_i^+ x_i(k)) \\
& \quad \times (h_i(x(k)) - \gamma_i^- x_i(k)) \le 0, \quad i = 1,2,\ldots,n,
\end{aligned} \tag{28}$$

which are equivalent to

$$\begin{aligned}
& \begin{pmatrix} x(k) \\ h(x(k)) \end{pmatrix}^T \begin{pmatrix} \gamma_i^-\gamma_i^+ e_i e_i^T & -\dfrac{\gamma_i^- + \gamma_i^+}{2} e_i e_i^T \\ * & e_i e_i^T \end{pmatrix} \\
& \times \begin{pmatrix} x(k) \\ h(x(k)) \end{pmatrix} \le 0,
\end{aligned} \tag{29}$$

where e_i denotes the unit column vector having one element on its ith row and zeros elsewhere.

Then from (29), for any matrices $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} > 0$, it follows that

$$\begin{pmatrix} x(k) \\ h(x(k)) \end{pmatrix}^T \begin{pmatrix} -\Gamma_1\Lambda & \Gamma_2\Lambda \\ * & -\Lambda \end{pmatrix} \begin{pmatrix} x(k) \\ h(x(k)) \end{pmatrix} \ge 0. \tag{30}$$

Note that, by Lemma 7, we get

$$\begin{aligned}
&E\left[\eta^T(k)P\eta(k)+2\eta^T(k)Px(k)+\tau^{*2}\eta^T(k)Z_1\eta(k)\right.\\
&\quad\left.+\tilde{\tau}\eta^T(k)Z_2\eta(k)\right]\\
&=E\left[\eta^T(k)\left(P+\tau^{*2}Z_1+\tilde{\tau}Z_2\right)\eta(k)+2\eta^T(k)Px(k)\right]\\
&=E\left[x^T(k+1)\left(P+\tau^{*2}Z_1+\tilde{\tau}Z_2\right)x(k+1)\right.\\
&\qquad-2x^T(k+1)\left(\tau^{*2}Z_1+\tilde{\tau}Z_2\right)x(k)\\
&\qquad\left.+x^T(k)\left(\tau^{*2}Z_1+\tilde{\tau}Z_2-P\right)x(k)\right]\\
&\le E\left[x^T(k+1)\left(P+2\tau^{*2}Z_1+2\tilde{\tau}Z_2\right)x(k+1)\right.\\
&\qquad\left.+x^T(k)\left(2\tau^{*2}Z_1+2\tilde{\tau}Z_2-P\right)x(k)\right]\\
&\le E\left[x^T(k+1)\lambda^*Ix(k+1)+x^T(k)\right.\\
&\qquad\left.\times\left(2\tau^{*2}Z_1+2\tilde{\tau}Z_2-P\right)x(k)\right]\\
&=E\left\{\left[(A+CK)x(k)+Bx(k-\tau(k))\right.\right.\\
&\qquad+D\sum_{i=-d(k)}^{-1}h(x(k+i))\\
&\qquad\left.+f(k,x(k),x(k-\tau(k)))+Cr(k)\right]^T\\
&\qquad\times\lambda^*I\left[(A+CK)x(k)+Bx(k-\tau(k))\right.\\
&\qquad\qquad+D\sum_{i=-d(k)}^{-1}h(x(k+i))\\
&\qquad\qquad\left.+f(k,x(k),x(k-\tau(k)))+Cr(k)\right]\\
&\qquad+[Gx(k)+Hx(k-\tau(k))]^T\\
&\qquad\times\lambda^*I[Gx(k)+Hx(k-\tau(k))]\\
&\qquad\left.+x^T(k)\left(2\tau^{*2}Z_1+2\tilde{\tau}Z_2-P\right)x(k)\right\}\\
&\le E\left\{\left[\left[(A+CK)x(k)+Bx(k-\tau(k))\right.\right.\right.\\
&\qquad+D\sum_{i=-d(k)}^{-1}h(x(k+i))\\
&\qquad\left.+f(k,x(k),x(k-\tau(k)))\right]^T\lambda^*I\\
&\qquad\times\left[(A+CK)x(k)+Bx(k-\tau(k))\right.\\
&\qquad\left.+D\sum_{i=-d(k)}^{-1}h(x(k+i))+f(k,x(k),x(k-\tau(k)))\right]\\
&\qquad+[Gx(k)+Hx(k-\tau(k))]^T\lambda^*I\\
&\qquad\times[Gx(k)+Hx(k-\tau(k))]\\
&\qquad+x^T(k)\left(2\tau^{*2}Z_1+2\tilde{\tau}Z_2-P\right)x(k)\\
&\qquad+\lambda^*x^T(A+CK)^T(A+CK)x(k)\\
&\qquad+\lambda^*x^T(k-\tau(k))B^TBx(k-\tau(k))\\
&\qquad+\lambda^*\left(\sum_{i=-d(k)}^{-1}h(x(k+i))\right)^TD^TD\\
&\qquad\times\left(\sum_{i=-d(k)}^{-1}h(x(k+i))\right)\\
&\qquad\left.+5\lambda^*\|C\|^2\|r\|_\infty^2\right\}.
\end{aligned}\tag{31}$$

Then from (18) to (31), we have

$$\begin{aligned}
E\Delta V(k)\le E&\left\{\xi^T(k)\left[\Xi'+\left(\sqrt{2}X,0,0,0,0,0,0,0\right)^T\frac{1}{\lambda^*}\right.\right.\\
&\left.\left.\times\left(\sqrt{2}X,0,0,0,0,0,0,0\right)\right]\xi(k)\right\}\\
&+\rho\|r\|_\infty^2,
\end{aligned}\tag{32}$$

where

$$\begin{aligned}
&\Xi'_{ij}=\Xi_{ij},\quad i,j=1,2,\ldots,8,\qquad\rho=5\lambda^*\|D\|^2,\\
&\xi^T(k)=\left[x^T(k),x^T(k-\tau(k)),x\left(k-\tau^*\right),x\left(k-d^*\right),\right.\\
&\qquad\left.\theta^T(k),h^T(x(k)),\sum_{-d(k)}^{-1}h^T(x(k+i)),f^T\right].
\end{aligned}\tag{33}$$

If the LMI (14) holds, by using Lemma 8, it follows that there exists a sufficient small positive $\varepsilon>0$, such that

$$E\Delta V(k)\le-\varepsilon E\|x(k)\|^2+\rho\|r\|_\infty^2.\tag{34}$$

It is easy to derive that

$$\begin{aligned}
EV(k)\le\mu_1E\|x(k)\|^2&+\mu_2\sum_{i=k-\tau_2}^{k-1}E\|x(i)\|^2\\
&+\mu_3\sum_{i=k-d_2}^{k-1}E\|x(i)\|^2,
\end{aligned}\tag{35}$$

with

$$\begin{aligned}\mu_1 &= \lambda_{\max}(P),\\ \mu_2 &= \lambda_{\max}(Q_1) + (a+1)\lambda_{\max}(Q_3)\\ &\quad + 2\lambda_{\max}(Q_4) + 4\tau^{*2}\lambda_{\max}(Z_1)\\ &\quad + 4(\tau_2-\tau_1)^2\lambda_{\max}(Z_2),\\ \mu_3 &= \lambda_{\max}(Q_2) + [d_2 + (d_2-d_1)\\ &\quad \times (d_2-d_1)]\\ &\quad \times \|\Gamma\|^2\lambda_{\max}(R).\end{aligned} \tag{36}$$

For any $\theta > 1$, it follows from (34) and (35) that

$$\begin{aligned}E&\left[\theta^{j+1}V(j+1) - \theta^j V(j)\right]\\ &= \theta^{j+1}E\Delta V(j) + \theta^j(\theta-1)EV(j)\\ &\le \theta^j\Bigg[(-\varepsilon\theta + (\theta-1)\mu_1)E\|x(j)\|^2\\ &\quad + (\theta-1)\mu_2\sum_{i=j-\tau_2}^{j-1}E\|x(i)\|^2 + \rho\theta\|r\|_\infty^2\\ &\quad + (\theta-1)\mu_3\sum_{i=j-d_2}^{j-1}E\|x(i)\|^2\Bigg].\end{aligned} \tag{37}$$

Summing up both sides of (37) from 0 to $k-1$, we can obtain

$$\begin{aligned}\theta^k &EV(k) - EV(0)\\ &\le (\mu_1(\theta-1) - \varepsilon\theta)\sum_{j=0}^{k-1}\theta^j E\|x(j)\|^2\\ &\quad + \mu_2(\theta-1)\sum_{j=0}^{k-1}\sum_{i=j-\tau_2}^{j-1}\theta^j E\|x(i)\|^2\\ &\quad + \mu_3(\theta-1)\sum_{j=0}^{k-1}\sum_{i=j-d_2}^{j-1}\theta^j E\|x(i)\|^2\\ &\quad + \rho\sum_{j=0}^{k-1}\theta^{j+1}\|r\|_\infty^2.\end{aligned} \tag{38}$$

Also it is easy to compute that

$$\begin{aligned}&\sum_{j=0}^{k-1}\sum_{i=j-\tau_2}^{j-1}\theta^j E\|x(i)\|^2\\ &\le \left(\sum_{i=-\tau_2}^{-1}\sum_{j=0}^{i+\tau_2} + \sum_{i=0}^{k-1-\tau_2}\sum_{j=i+1}^{i+\tau_2} + \sum_{i=k-\tau_2}^{k-1}\sum_{j=i+1}^{k-1}\right)\\ &\quad\times \theta^j E\|x(i)\|^2\\ &\le \tau_2\theta^{\tau_2}\sup_{s\in[-\tau_2,0]}E\|x(s)\|^2 + \tau_2\theta^{\tau_2}\sum_{i=0}^{k-1}\theta^i E\|x(i)\|^2,\\ &\sum_{j=0}^{k-1}\sum_{i=j-d_2}^{j-1}\theta^j E\|x(i)\|^2\\ &\le \left(\sum_{i=-d_2}^{-1}\sum_{j=0}^{i+d_2} + \sum_{i=0}^{k-1-d_2}\sum_{j=i+1}^{i+d_2} + \sum_{i=k-d_2}^{k-1}\sum_{j=i+1}^{k-1}\right)\\ &\quad\times \theta^j E\|x(i)\|^2\\ &\le d_2\theta^{d_2}\sup_{s\in[-d_2,0]}E\|x(s)\|^2 + d_2\theta^{d_2}\\ &\quad\times \sum_{i=0}^{k-1}\theta^i E\|x(i)\|^2.\end{aligned} \tag{39}$$

Substituting (39) into (38) leads to

$$\begin{aligned}\theta^k &EV(k) - EV(0)\\ &\le \eta_1(\theta)\sup_{s\in[-\tau_2,0]}E\|x(s)\|^2 + \rho\sum_{j=0}^{k-1}\theta^{j+1}\|r\|_\infty^2\\ &\quad + \eta_2(\theta)\sum_{i=0}^{k-1}\theta^i E\|x(i)\|^2 + \eta_3(\theta)\sum_{i=0}^{k-1}\theta^i E\|x(i)\|^2,\end{aligned} \tag{40}$$

where $\eta_1(\theta) = \mu_2(\theta-1)\tau_2\theta^{\tau_2} + \mu_3(\theta-1)d_2\theta^{d_2}$, $\eta_2(\theta) = \mu_2(\theta-1)\tau_2\theta^{\tau_2} + \mu_1(\theta-1) - \varepsilon\theta$, $\eta_3(\theta) = \mu_3(\theta-1)d_2\theta^{d_2} + \mu_1(\theta-1) - \varepsilon\theta$.

Since $\eta_2(1) < 0$, $\eta_3(1) < 0$, there must exist a positive $\theta_0 > 1$ such that $\eta_2(\theta_0) < 0$, $\eta_3(\theta_0) < 0$. Then we have

$$\begin{aligned}EV&(k)\\ &\le \eta_1(\theta_0)\left(\frac{1}{\theta_0}\right)^k\sup_{s\in[-\tau_2,0]}E\|x(s)\|^2\\ &\quad + \left(\frac{1}{\theta_0}\right)^k EV(0) + \rho\sum_{j=0}^{k-1}\frac{1}{\theta_0^{k-j-1}}\|r\|_\infty^2\\ &\quad + \eta_2(\theta_0)\sum_{i=0}^{k-1}\frac{1}{\theta_0^{k-i}}E\|x(i)\|^2 + \eta_3(\theta_0)\sum_{i=0}^{k-1}\frac{1}{\theta_0^{k-i}}E\|x(i)\|^2\\ &\le (\eta_1(\theta_0) + \mu_1 + \mu_2\tau_2 + \mu_3 d_2)\\ &\quad \times \sup_{s\in[-\max\{\tau_2,d_2\},0]}E\|x(s)\|^2 + \frac{\rho}{\theta_0-1}\|r\|_\infty^2.\end{aligned} \tag{41}$$

On the other hand, by (16) we can get

$$EV(k) \ge \lambda_{\min}(P)E\|x(k)\|^2. \tag{42}$$

Combining (41) with (42), we have

$$\begin{aligned}E\|x(k)\|^2 &\le \frac{\eta_1(\theta_0) + \mu_1 + \mu_2\tau_2 + \mu_3 d_2}{\lambda_{\min}(P)}\\ &\quad \times \sup_{s\in[-\max\{\tau_2,d_2\},0]}E\|x(s)\|^2\\ &\quad + \frac{1}{\lambda_{\min}(P)}\frac{\rho}{\theta_0-1}\|r\|_\infty^2.\end{aligned} \tag{43}$$

Thus,

$$\mathbb{E}\|y(k)\|^2 \le \|M\|^2 E\|x(k)\|^2 \le N_1 + N_2\|r\|_\infty^2, \quad (44)$$

where $N_1 = \|M\|^2((\eta_1(\theta_0) + \mu_1 + \mu_2\tau_2 + \mu_3 d_2)/\lambda_{\min}(P)) \sup_{s\in[-\max\{\tau_2,d_2\},0]} E\|x(s)\|^2$, $N_2 = (1/\lambda_{\min}(P))(\rho/(\theta_0 - 1))\|M\|^2$. By Definition 5, the nonlinear discrete-time stochastic control system (1) is BIBO stability in mean square. This completes the proof. □

If the stochastic term $\omega(K)$ is removed in (1), then the following results can be obtained.

Theorem 10. *For given positive integers τ_1, τ_2, d_1, and d_2, under Assumption 2, the nonlinear discrete-time stochastic control system (1) with the controller (6) is BIBO stabilization in mean square, if there exist symmetric positive-definite matrix P, R, Q_i, $i = 1, 2, \ldots, 5$, Z_1, Z_2, and X with appropriate dimensional positive-definite diagonal matrices Λ and two positive constants ζ and λ^* such that the following two LMIs hold:*

$$P + 2\left(\tau^{*2} Z_1 + \tilde{\tau}^2 Z_2\right) \le \lambda^* I,$$

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} & Z_1 & 0 & 0 & \Gamma_2\Lambda & \Xi_{17} & \Xi_{18} & \sqrt{2}X \\ * & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & B^T D & B^T & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & \lambda^* D^T & 0 \\ * & * & * & * & * & * & * & -\zeta I & 0 \\ * & * & * & * & * & * & * & * & -\lambda^* I \end{pmatrix} \le 0, \quad (45)$$

where

$$\begin{aligned}
\Xi_{11} &= Q_1 + Q_2 + Q_3 + a(Q_3 + Q_4) - Z_1 \\
&\quad + Q_5 - \Gamma_1\Lambda + \zeta I + 2\tau^{*2} Z_1 + 2\tilde{\tau}^2 Z_2 \\
&\quad + 2\lambda^*\alpha_1 I + \alpha_1\zeta I - P + 2\lambda^* A^T A \\
&\quad - 2A^T X - 2X^T A, \\
\Xi_{22} &= 2\lambda^* B^T B - Q_4 - 2Z_2 - \beta Z_2 \\
&\quad + 2\lambda^*\alpha_2 I + \alpha_2\zeta I - (1-\beta) Z_2, \\
\Xi_{12} &= \lambda^* A^T B - X^T B, \\
\Xi_{17} &= \lambda^* A^T D - X^T D, \\
\Xi_{18} &= \lambda^* A^T - X^T, \\
\Xi_{25} &= Z_2 + (1-\beta) Z_2, \\
\Xi_{23} &= Z_2 + \beta Z_2, \\
\Xi_{33} &= -Q_1 - Z_1 - Z_2 - \beta Z_2, \\
\Xi_{55} &= -Q_5 - Z_2 - (1-\beta) Z_2, \\
\Xi_{66} &= bR - \Lambda, \\
\Xi_{77} &= 2\lambda^* D^T D - \frac{1}{c} R, \\
X &= -\lambda^* CK.
\end{aligned} \quad (46)$$

Proof. The proof is straightforward and hence omitted. □

Corollary 11. *System (1) is also stabilization in mean square when all the conditions in Theorems 9 and 10 are satisfied, if the bounded input $r(t) = 0$ in (6).*

Remark 12. In this paper, a novel BIBO stability criterion for system (1) is derived by checking the variation of derivatives of the Lyapunov-Krasovskii functionals for each subinterval. It is different from [27–32], which checked the variation of the Lyapunov functional in the whole variation interval of the delay.

Remark 13. The BIBO stabilization criteria for discrete-time systems have been investigated in the recently reported paper [28]. However, the stochastic disturbances and nonlinear perturbations have not been taken into account in the control systems. In [28], the time delay is constant time, which is a special case of this paper when $\tau_1 = \tau_2$.

Remark 14. The mean square stabilization conditions in Theorem 9 in this paper depend on the time-delays upper bounds and the lower bounds, time-delays interval, and time-delay interval segmentation point and relate to the delays themselves.

Remark 15. In [33], the time-delay interval is divided into two equal subintervals; the interval segmentation point is midpoint. In this paper, the time-delay interval is divided into two any subintervals; the interval segmentation point is any point in the time-delay interval.

4. An Example

In this section, a numerical example will be presented to show the validity of the main results derived in Section 3.

Table 1: For $\tau(k) = 1, 2, 3, 4, 5, \beta$.

$\tau(k)$	1	2	3	4	5
β	0	1	1/2	1/2	0

Example 1. As a simple application of Theorem 9, consider the stochastic control system (1) with the control law (6); the parameters are given by

$$A = \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.1 & 0.1 \\ -0.1 & 0.1 \end{pmatrix},$$
$$C = \begin{pmatrix} 0.1 & 0.1 \\ 0.5 & 0.3 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.1 & 0.1 \\ 0 & 0.2 \end{pmatrix}, \tag{47}$$

$G = 0.001I$, $H = 0.02I$, $f = [0.1x(k), \sqrt{0.2}x(k-\tau(k))]^T$, $h_1(s) = \sin(0.2s) - 0.6\cos(s)$, $h_2(s) = \tanh(-0.4s)$, $\tau_1 = 1$, $\tau_2 = 5$, $d_1 = 2$, $d_2 = 7$.

It is easy to verify that $a = 5$, $\tau^* = 3$, $d^* = 4$, $\tilde{\tau} = 2$, and

$$\Gamma_1 = \begin{pmatrix} -0.64 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 0 & 0 \\ 0 & -0.2 \end{pmatrix},$$
$$\tau^* = \frac{\tau_1+\tau_2}{2} - \frac{\min\{(-1)^{\tau_1+\tau_2}, 0\}}{2},$$
$$d^* = \frac{\tau_1+\tau_2}{2} + \frac{\min\{(-1)^{\tau_1+\tau_2}, 0\}}{2}, \tag{48}$$
$$b = \begin{cases} 9, & d_1 \le d(k) \le d^*, \\ 20, & d^* < d(k) \le d_2, \end{cases}$$
$$c = \begin{cases} 4, & d_1 \le d(k) \le d^*, \\ 7, & d^* < d(k) \le d_2. \end{cases}$$

Meanwhile, the corresponding values of β for various $\tau(k)$ are listed in Table 1.

By using the MATLAB LMI Toolbox, we solve LMIs (13), (14) and obtain six groups of feasible solutions; we list one case as follows.

When $\beta = 1$, $b = 9$, $c = 4$,

$$P = \begin{pmatrix} 145.0684 & -3.2651 \\ -3.2651 & 147.7799 \end{pmatrix}, \qquad Z_1 = \begin{pmatrix} 0.0478 & 0.0298 \\ 0.0298 & 0.0230 \end{pmatrix},$$
$$Z_2 = \begin{pmatrix} 0.3443 & 0.2142 \\ 0.2142 & 0.1663 \end{pmatrix}, \qquad Q_1 = \begin{pmatrix} 2.8401 & 1.7808 \\ 1.7808 & 1.3624 \end{pmatrix},$$
$$Q_2 = \begin{pmatrix} 3.7464 & 2.3462 \\ 2.3462 & 1.7985 \end{pmatrix}, \qquad Q_3 = \begin{pmatrix} 0.2731 & 0.1713 \\ 0.1713 & 0.1309 \end{pmatrix},$$
$$Q_4 = \begin{pmatrix} 15.3027 & -6.8645 \\ -6.8645 & 15.5746 \end{pmatrix}, \qquad Q_5 = \begin{pmatrix} 3.1500 & 1.9740 \\ 1.9740 & 1.5118 \end{pmatrix},$$
$$R = \begin{pmatrix} 128.2646 & -92.1100 \\ -92.1100 & 239.0910 \end{pmatrix}, \qquad N = \begin{pmatrix} 0.4512 & 0 \\ 0 & 34.3341 \end{pmatrix},$$
$$X = \begin{pmatrix} 13.3495 & 3.1121 \\ 3.1121 & 0.7692 \end{pmatrix}, \qquad K = \begin{pmatrix} -1.4212 & -0.3303 \\ 0.5268 & 0.1218 \end{pmatrix}, \tag{49}$$

and $\zeta = 82.0259$, $\lambda^* = 150.2996$.

The system (1) exhibits stabilization in mean square behavior as shown in Figure 1.

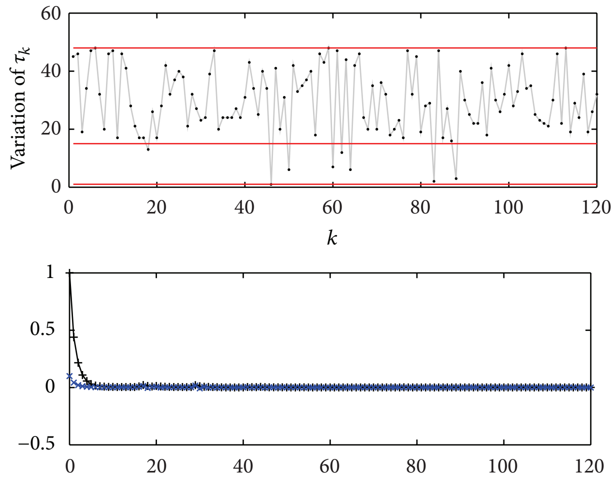

Figure 1

5. Conclusions

In this paper, we have derived some conditions for the BIBO stabilization in mean square for a class of discrete-time stochastic control systems with mixed time-varying delays. The results have been obtained by constructing a novel Lyapunov-Krasovskii function. The conditions are expressed in the forms of linear matrix inequalities, which can be easily checked by using MATLAB LMI Toolbox. A numerical example is given to illustrate the validity of the obtained results.

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