Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 701910, 18 pages
doi:10.1155/2012/701910

Research Article

On Generalized Weakly G-Contractive Mappings in Partially Ordered G-Metric Spaces

A. Razani and V. Parvaneh

Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran

Correspondence should be addressed to A. Razani, razani@ipm.ac.ir

Received 3 August 2012; Accepted 6 September 2012

Academic Editor: Naseer Shahzad

The aim of this paper is to present some coincidence and common fixed point results for generalized weakly G-contractive mappings in the setup of partially ordered G-metric space. We also provide an example to illustrate the results presented herein. As an application of our results, periodic points of weakly G-contractive mappings are obtained.

1. Introduction and Mathematical Preliminaries

The concept of a generalized metric space, or a G-metric space, was introduced by Mustafa et al. [1]. In recent years, many authors have obtained different fixed point theorems for mappings satisfying various contractive conditions on G-metric spaces. For a survey of fixed point theory, its applications, comparison of different contractive conditions, and related topics in G-metric spaces we refer the reader to [1–14] and the references mentioned therein.

Definition 1.1 (G-metric space [1]). Let X be a nonempty set and $G : X \times X \times X \to R^+$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if and only if $x = y = z$;
(G2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$;
(G3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$;
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$, (symmetry in all three variables);
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality).

Then, the function G is called a G-metric on X and the pair (X, G) is called a G-metric space.

Definition 1.2 (see [1]). Let (X, G) be a G-metric space and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$ and one says that the sequence $\{x_n\}$ is G-convergent to x. Thus, if $x_n \to x$ in a G-metric space (X, G), then for any $\varepsilon > 0$, there exists a positive integer N such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq N$.

Definition 1.3 (see [1]). Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if for every $\varepsilon > 0$, there is a positive integer N such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq N$, that is, if $G(x_n, x_m, x_l) \to 0$, as $n, m, l \to \infty$.

Lemma 1.4 (see [1]). *Let (X, G) be a G-metric space. Then, the following are equivalent:*

(1) *$\{x_n\}$ is G-convergent to x.*

(2) $G(x_n, x_n, x) \to 0$, *as* $n \to \infty$.

(3) $G(x_n, x, x) \to 0$, *as* $n \to \infty$.

(4) $G(x_m, x_n, x) \to 0$, *as* $m, n \to \infty$.

Lemma 1.5 (see [15]). *If (X, G) is a G-metric space, then $\{x_n\}$ is a G-Cauchy sequence if and only if for every $\varepsilon > 0$, there exists a positive integer N such that $G(x_n, x_m, x_m) < \varepsilon$, for all $m > n \geq N$.*

Definition 1.6 (see [1]). A G-metric space (X, G) is said to be G-complete (or complete G-metric space) if every G-Cauchy sequence in (X, G) is convergent in X.

Definition 1.7 (see [1]). Let (X, G) and (X', G') be two G-metric spaces. Then a function $f : X \to X'$ is G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x, that is, whenever $\{x_n\}$ is G-convergent to x, $\{f(x_n)\}$ is G-convergent to $f(x)$.

The concept of an altering distance function was introduced by Khan et al. [16] as follows.

Definition 1.8. The function $\psi : [0, \infty) \to [0, \infty)$ is called an altering distance function, if the following properties are satisfied.

(1) ψ is continuous and nondecreasing.

(2) $\psi(t) = 0$ if and only if $t = 0$.

In [5], Aydi et al. established some common fixed point results for two self-mappings f and g on a generalized metric space X. They presented the following definitions.

Definition 1.9 (see [5]). Let (X, G) be a G-metric space and $f, g : X \to X$ be two mappings. We say that f is a generalized weakly G-contraction mapping of type A with respect to g if for all $x, y, z \in X$, the following inequality holds:

$$\psi(G(fx, fy, fz)) \leq \psi\left(\frac{G(gx, fy, fy) + G(gy, fz, fz) + G(gz, fx, fx)}{3}\right) - \varphi(G(gx, fy, fy), G(gy, fz, fz), G(gz, fx, fx)), \tag{1.1}$$

where

(1) ψ is an altering distance function;

(2) $\varphi : [0, \infty)^3 \to [0, \infty)$ is a continuous function with $\varphi(t, s, u) = 0$ if and only if $t = s = u = 0$.

Definition 1.10 (see [5]). Let (X, G) be a G-metric space and $f, g : X \to X$ be given mappings. We say that f is a generalized weakly G-contraction mapping of type B with respect to g if for all $x, y, z \in X$, the following inequality holds:

$$\begin{aligned}\psi(G(fx, fy, fz)) \le{}& \psi\left(\frac{G(gx, gx, fy) + G(gy, gy, fz) + G(gz, gz, fx)}{3}\right)\\ &- \varphi(G(gx, gx, fy), G(gy, gy, fz), G(gz, gz, fx)),\end{aligned} \tag{1.2}$$

where

(1) ψ is an altering distance function;

(2) $\varphi : [0, \infty)^3 \to [0, \infty)$ is a continuous function with $\varphi(t, s, u) = 0$ if and only if $t = s = u = 0$.

Note that the concept of a generalized weakly G-contraction is the extension of the concept of weakly C-contraction which has been defined by Choudhury in [17]. For more details on weakly C-contractive mappings we refer the reader to [18, 19].

Definition 1.11 (see [20]). Let $(X, \preceq)$ be a partially ordered set. A mapping f is called a dominating map on X if $x \preceq fx$ for each x in X.

Example 1.12 (see [20]). Let $X = [0, 1]$ be endowed with the usual ordering. Let $f : X \to X$ be defined by $fx = x^{1/3}$. Then, $x \le x^{1/3} = fx$ for all $x \in X$. Thus, f is a dominating map.

Example 1.13 (see [20]). Let $X = [0, \infty)$ be endowed with the usual ordering. Let $f : X \to X$ be defined by $fx = \sqrt[n]{x}$ for $x \in [0, 1)$ and $fx = x^n$ for $x \in [1, \infty)$, for any $n \in \mathbb{N}$. Then, for all $x \in X$, $x \le fx$; that is, f is a dominating map.

A subset W of a partially ordered set X is said to be well ordered if every two elements of W be comparable [20].

The following definition is Definition 2.5 of [21], but in the setup of partially ordered G-metric spaces.

Definition 1.14. Let $(X, \preceq, G)$ be a partially ordered G-metric space. We say that X is regular if and only if the following hypothesis holds.

For any nondecreasing sequence $\{x_n\}$ in X such that $x_n \to z$ as $n \to \infty$, it follows that $x_n \preceq z$ for all $n \in \mathbb{N}$.

Jungck in [22] introduced the following definition.

Definition 1.15 (see [22]). Let (X, d) be a metric space and $f, g : X \to X$. The pair (f, g) is said to be compatible if and only if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Let X be a nonempty set and $f : X \to X$ be a given mapping. For every $x \in X$, let $f^{-1}(x) = \{u \in X \mid fu = x\}$.

Definition 1.16 (see [21]). Let $(X, \preceq)$ be a partially ordered set and $f, g, h : X \to X$ are given mappings such that $fX \subseteq hX$ and $gX \subseteq hX$. We say that f and g are weakly increasing with respect to h if and only if for all $x \in X$, we have

$$\begin{aligned} fx &\preceq gy, \quad \forall y \in h^{-1}(fx), \\ gx &\preceq fy, \quad \forall y \in h^{-1}(gx). \end{aligned} \tag{1.3}$$

If $f = g$, we say that f is weakly increasing with respect to h.

If $h = I$ (the identity mapping on X), then the above definition reduces to the weakly increasing mapping [23] (also see [21, 24]).

Definition 1.17. Let (X, G) be a G-metric space and $f, g : X \to X$. The pair (f, g) is said to be compatible if and only if $\lim_{n\to\infty} G(fgx_n, fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Note that the concept of compatibility in a G-metric space has been defined by Kumar in [25] (Definition 2.1). In the above definition we only modify his definition, using the fact that $G(x, y, y) \le 2G(x, x, y)$, for all $x, y \in X$.

The aim of this paper is to prove some coincidence and common fixed point theorems for nonlinear weakly G-contractive mappings in partially ordered G-metric spaces.

2. Main Results

From now, we assume

$$\begin{aligned} \Phi = \Big\{ \varphi \mid \varphi : [0, \infty)^3 \longrightarrow [0, \infty) \text{ is a continuous} \\ \text{function such that } \varphi(x, y, z) = 0 \Longleftrightarrow x = y = z = 0 \Big\}. \end{aligned} \tag{2.1}$$

Our first result is the following.

Theorem 2.1. *Let $(X, \preceq, G)$ be a partially ordered complete G-metric space. Let $f, g : X \to X$ be two mappings such that $f(X) \subseteq g(X)$; f is weakly increasing with respect to g and*

$$\begin{aligned} \psi(G(fx, fy, fz)) \le \psi\left(\frac{G(gx, fy, fy) + G(gy, fz, fz) + G(gz, fx, fx)}{3} \right) \\ - \varphi(G(gx, fy, fy), G(gy, fz, fz), G(gz, fx, fx)) \end{aligned} \tag{2.2}$$

for every $x, y, z \in X$ such that $gx \preceq gy \preceq gz$, where $\psi : [0, \infty) \to [0, \infty)$ is an altering distance function and $\varphi \in \Phi$. Then f and g have a coincidence point in X provided that f and g are continuous and the pair (f, g) is compatible.

Proof. Let $x_0 \in X$ be an arbitrary point. Since $f(X) \subseteq g(X)$, we can construct a sequence $\{z_n\}$ defined by: $z_n = gx_n = fx_{n-1}$, for all $n \ge 0$.

Now, since $x_1 \in g^{-1}(fx_0)$ and $x_2 \in g^{-1}(fx_1)$, as f is weakly increasing with respect to g, we obtain

$$gx_1 = fx_0 \preceq fx_1 = gx_2 \preceq fx_2 = gx_3. \tag{2.3}$$

Continuing this process, we get:

$$gx_1 \preceq gx_2 \preceq gx_3 \preceq \cdots \preceq gx_n \preceq gx_{n+1} \preceq \cdots . \tag{2.4}$$

We complete the proof in three steps.

Step I. We will prove that $\lim_{n\to\infty} G(z_n, z_{n+1}, z_{n+1}) = 0$.

Since $gx_{n-1} \preceq gx_n$, using (2.2) we obtain that

$$\begin{aligned}
\psi(G(z_n, z_{n+1}, z_{n+1})) &= \psi(G(fx_{n-1}, fx_n, fx_n)) \\
&\le \psi\left(\frac{G(gx_{n-1}, fx_n, fx_n) + G(gx_n, fx_n, fx_n) + G(gx_n, fx_{n-1}, fx_{n-1})}{3}\right) \\
&\quad - \varphi(G(gx_{n-1}, fx_n, fx_n), G(gx_n, fx_n, fx_n), G(gx_n, fx_{n-1}, fx_{n-1})) \\
&= \psi\left(\frac{G(z_{n-1}, z_{n+1}, z_{n+1}) + G(z_n, z_{n+1}, z_{n+1}) + G(z_n, z_n, z_n)}{3}\right) \\
&\quad - \varphi(G(z_{n-1}, z_{n+1}, z_{n+1}), G(z_n, z_{n+1}, z_{n+1}), G(z_n, z_n, z_n)) \\
&\le \psi\left(\frac{G(z_{n-1}, z_n, z_n) + 2G(z_n, z_{n+1}, z_{n+1})}{3}\right) \\
&\quad - \varphi(G(z_{n-1}, z_{n+1}, z_{n+1}), G(z_n, z_{n+1}, z_{n+1}), G(z_n, z_n, z_n)) \\
&\le \psi\left(\frac{G(z_{n-1}, z_n, z_n) + 2G(z_n, z_{n+1}, z_{n+1})}{3}\right).
\end{aligned} \tag{2.5}$$

Since ψ is a nondecreasing function, from (2.5), we have

$$\begin{aligned}
G(z_n, z_{n+1}, z_{n+1}) &\le \frac{G(z_{n-1}, z_{n+1}, z_{n+1}) + G(z_n, z_{n+1}, z_{n+1})}{3} \\
&\le \frac{G(z_{n-1}, z_n, z_n) + 2G(z_n, z_{n+1}, z_{n+1})}{3}.
\end{aligned} \tag{2.6}$$

Hence, we conclude that $\{G(z_n, z_{n+1}, z_{n+1})\}$ is a nondecreasing sequence of nonnegative real numbers. Thus, there is an $r \ge 0$ such that

$$\lim_{n\to\infty} G(z_n, z_{n+1}, z_{n+1}) = r. \tag{2.7}$$

Letting $n \to \infty$ in (2.6), we get that

$$r \leq \frac{\lim_{n\to\infty} G(z_{n-1}, z_{n+1}, z_{n+1}) + r}{3} \leq r, \tag{2.8}$$

that is,

$$\lim_{n\to\infty} G(z_{n-1}, z_{n+1}, z_{n+1}) = 2r. \tag{2.9}$$

Again, from (2.5) we have

$$\begin{aligned}\psi(G(z_n, z_{n+1}, z_{n+1})) = \psi\left(\frac{G(z_{n-1}, z_{n+1}, z_{n+1}) + G(z_n, z_{n+1}, z_{n+1}) + G(z_n, z_n, z_n)}{3}\right) \\ - \varphi(G(z_{n-1}, z_{n+1}, z_{n+1}), G(z_n, z_{n+1}, z_{n+1}), G(z_n, z_n, z_n)).\end{aligned} \tag{2.10}$$

Letting $n \to \infty$ and using (2.7), (2.9), and the continuities of ψ and φ, we get $\psi(r) \leq \psi((2r + r + 0)/3) - \varphi(2r, r, 0)$, and hence $\varphi(2r, r, 0) = 0$. This gives us that

$$\lim_{n\to\infty} G(z_n, z_{n+1}, z_{n+1}) = 0, \tag{2.11}$$

from our assumptions about φ.

Step II. We will show that $\{z_n\}$ is a G-Cauchy sequences in X. So, we will show that for every $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $m, n \geq k$,

$$G(z_m, z_n, z_n) < \varepsilon. \tag{2.12}$$

Suppose the above statement is false. Then, there exists $\varepsilon > 0$ for which we can find subsequences $\{z_{m(k)}\}$ and $\{z_{n(k)}\}$ of $\{z_n\}$ such that $n(k) > m(k) > k$ and

$$G\left(z_{m(k)}, z_{n(k)}, z_{n(k)}\right) \geq \varepsilon, \tag{2.13}$$

where $n(k)$ is the smallest index with this property, that is,

$$G\left(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}\right) < \varepsilon. \tag{2.14}$$

From rectangle inequality,

$$G\left(z_{m(k)}, z_{n(k)}, z_{n(k)}\right) \leq G\left(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}\right) + G\left(z_{n(k)-1}, z_{n(k)}, z_{n(k)}\right). \tag{2.15}$$

Making $k \to \infty$ in (2.15), from (2.11), (2.13), and (2.14) we conclude that

$$\lim_{k\to\infty} G\left(z_{m(k)}, z_{n(k)}, z_{n(k)}\right) = \varepsilon. \tag{2.16}$$

Again, from rectangle inequality,

$$\begin{aligned} G(z_{m(k)}, z_{n(k)}, z_{n(k)+1}) &\leq G(z_{m(k)}, z_{n(k)}, z_{n(k)}) + G(z_{n(k)}, z_{n(k)}, z_{n(k)+1}) \\ &\leq G(z_{m(k)}, z_{n(k)}, z_{n(k)}) + 2G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}), \\ G(z_{n(k)}, z_{n(k)}, z_{m(k)}) &\leq G(z_{n(k)}, z_{m(k)}, z_{n(k)+1}). \end{aligned} \tag{2.17}$$

Hence in (2.17), if $k \to \infty$, using (2.11), and (2.16), we have

$$\lim_{k\to\infty} G(z_{m(k)}, z_{n(k)}, z_{n(k)+1}) = \varepsilon. \tag{2.18}$$

On the other hand,

$$G(z_{m(k)}, z_{n(k)+1}, z_{n(k)+1}) \leq G(z_{m(k)}, z_{n(k)}, z_{n(k)}) + G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}), \tag{2.19}$$

and

$$G(z_{n(k)}, z_{n(k)+1}, z_{m(k)}) \leq G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)+1}, z_{n(k)+1}, z_{m(k)}). \tag{2.20}$$

Hence in (2.19) and (2.20), if $k \to \infty$, from (2.11), (2.16) and (2.18) we have

$$\lim_{k\to\infty} G(z_{m(k)}, z_{n(k)+1}, z_{n(k)+1}) = \varepsilon. \tag{2.21}$$

In a similar way, we have

$$\begin{aligned} G(z_{m(k)+1}, z_{n(k)}, z_{n(k)+1}) &\leq G(z_{m(k)+1}, z_{m(k)}, z_{m(k)}) + G(z_{m(k)}, z_{n(k)}, z_{n(k)+1}) \\ &\leq 2G(z_{m(k)}, z_{m(k)+1}, z_{m(k)+1}) + G(z_{m(k)}, z_{n(k)}, z_{n(k)+1}), \\ G(z_{m(k)}, z_{n(k)}, z_{n(k)+1}) &\leq G(z_{m(k)}, z_{m(k)+1}, z_{m(k)+1}) + G(z_{m(k)+1}, z_{n(k)}, z_{n(k)+1}), \end{aligned} \tag{2.22}$$

and therefore, from (2.22) by taking limit when $k \to \infty$, using (2.11) and (2.18), we get that

$$\lim_{k\to\infty} G(z_{m(k)+1}, z_{n(k)}, z_{n(k)+1}) = \varepsilon. \tag{2.23}$$

Also,

$$\begin{aligned} G(z_{m(k)+1}, z_{n(k)+1}, z_{n(k)+1}) &\leq G(z_{m(k)+1}, z_{n(k)+1}, z_{n(k)}), \\ G(z_{m(k)+1}, z_{n(k)}, z_{n(k)+1}) &\leq G(z_{m(k)+1}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)+1}, z_{n(k)+1}, z_{n(k)}). \end{aligned} \tag{2.24}$$

So, from (2.11), (2.23), and (2.24), we have

$$\lim_{k\to\infty} G(z_{m(k)+1}, z_{n(k)+1}, z_{n(k)+1}) = \varepsilon. \tag{2.25}$$

Finally,

$$\begin{aligned}
G(z_{n(k)}, z_{m(k)+1}, z_{m(k)+1}) &\leq G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)+1}, z_{m(k)+1}, z_{m(k)+1}), \\
G(z_{n(k)+1}, z_{m(k)+1}, z_{m(k)+1}) &\leq G(z_{n(k)+1}, z_{n(k)}, z_{n(k)}) + G(z_{n(k)}, z_{m(k)+1}, z_{m(k)+1}) \\
&\leq G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)}, z_{m(k)+1}, z_{m(k)+1}).
\end{aligned} \tag{2.26}$$

Hence in (2.26), if $k \to \infty$ and using (2.11) and (2.25), we have

$$\lim_{k\to\infty} G(z_{m(k)}, z_{n(k)+1}, z_{n(k)+1}) = \varepsilon. \tag{2.27}$$

Since $gx_{m(k)} \preceq gx_{n(k)} \preceq gx_{n(k)}$, putting $x = x_{m(k)}$, $y = x_{n(k)}$, and $z = x_{n(k)}$ in (2.2), for all $k \geq 0$, we have

$$\begin{aligned}
&\psi(G(z_{m(k)+1}, z_{n(k)+1}, z_{n(k)+1})) \\
&\quad = \psi(G(fx_{m(k)}, fx_{n(k)}, fx_{n(k)})) \\
&\quad \leq \psi\left(\frac{G(gx_{m(k)}, fx_{n(k)}, fx_{n(k)}) + G(gx_{n(k)}, fx_{n(k)}, fx_{n(k)}) + G(gx_{n(k)}, fx_{m(k)}, fx_{m(k)})}{3}\right) \\
&\qquad - \varphi(G(gx_{m(k)}, fx_{n(k)}, fx_{n(k)}), G(gx_{n(k)}, fx_{n(k)}, fx_{n(k)}), G(gx_{n(k)}, fx_{m(k)}, fx_{m(k)})) \\
&\quad \leq \psi\left(\frac{G(z_{m(k)}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}) + G(z_{n(k)}, z_{m(k)+1}, z_{m(k)+1})}{3}\right) \\
&\qquad - \varphi(G(z_{m(k)}, z_{n(k)+1}, z_{n(k)+1}), G(z_{n(k)}, z_{n(k)+1}, z_{n(k)+1}), G(z_{n(k)}, z_{m(k)+1}, z_{m(k)+1})).
\end{aligned} \tag{2.28}$$

Now, if $k \to \infty$ in (2.28), from (2.11), (2.21), (2.25), and (2.27), we have

$$\psi(\varepsilon) \leq \psi\left(\frac{2\varepsilon}{3}\right) - \varphi(\varepsilon, 0, \varepsilon). \tag{2.29}$$

Hence, $\varepsilon = 0$ which is a contradiction. Consequently, $\{z_n\}$ is G-Cauchy.

Step III. We will show that f and g have a coincidence point.

Since $\{gx_n\}$ is a G-Cauchy sequence in the complete G-metric space X, there exists $z \in X$ such that

$$\lim_{n\to\infty} G(z_n, z_n, z) = \lim_{n\to\infty} G(gx_n, gx_n, z) = \lim_{n\to\infty} G(fx_n, fx_n, z) = 0. \tag{2.30}$$

From (2.30) and the continuity of g, we get

$$\lim_{n\to\infty} G(gz_n, gz_n, gz) = \lim_{n\to\infty} G(g(gx_n), g(gx_n), gz) = 0. \tag{2.31}$$

By the rectangle inequality, we have

$$\begin{aligned} G(gz, fz, fz) &\le G(gz, ggx_{n+1}, ggx_{n+1}) + G(gfx_n, fz, fz) \\ &\le G(gz, ggx_{n+1}, ggx_{n+1}) + G(gfx_n, fgx_n, fgx_n) + G(fgx_n, fz, fz). \end{aligned} \tag{2.32}$$

From (2.30), as $n \to \infty$, we have

$$gx_n \longrightarrow z, \qquad fx_n \longrightarrow z. \tag{2.33}$$

Since the pair (f, g) is compatible, this implies that

$$\lim_{n\to\infty} G(gfx_n, fgx_n, fgx_n) = 0. \tag{2.34}$$

Now, from the continuity of f and (2.30), we have

$$\lim_{n\to\infty} G(fz_n, fz, fz) = 0. \tag{2.35}$$

Combining (2.31), (2.32), and (2.34) and letting $n \to \infty$ in (2.35), we obtain

$$G(gz, fz, fz) \le 0, \tag{2.36}$$

which implies that $fz = gz$, that is, z is a coincidence point of f and g. □

In the following theorem, we will omit the continuity of f and g, and the compatibility of the pair (f, g).

Theorem 2.2. *Let $(X, \preceq, G)$ be a partially ordered G-metric space. Let $f, g : X \to X$ be two mappings such that $f(X) \subseteq g(X)$; f is weakly increasing with respect to g and*

$$\begin{aligned} \psi(G(fx, fy, fz)) \le \psi&\left(\frac{G(gx, fy, fy) + G(gy, fz, fz) + G(gz, fx, fx)}{3}\right) \\ &- \varphi(G(gx, fy, fy), G(gy, fz, fz), G(gz, fx, fx)), \end{aligned} \tag{2.37}$$

for every $x, y, z \in X$ such that $gx \preceq gy \preceq gz$, where $\psi : [0,\infty) \to [0,\infty)$ is an altering distance function and $\varphi \in \Phi$. Then, f and g have a coincidence point in X if X is regular and $g(X)$ is a G-complete subset of (X, G).

Proof. Following the proof of Theorem 2.1, there exists $z \in X$ such that

$$\lim_{n\to\infty} G(z_n, z_n, z) = \lim_{n\to\infty} G(gx_n, gx_n, z) = \lim_{n\to\infty} G(fx_n, fx_n, z) = 0. \tag{2.38}$$

Since $g(X)$ is G-complete and $\{z_n\} \subseteq g(X)$, we have $z \in g(X)$ and hence there exists $u \in X$ such that $z = gu$ and

$$\lim_{n\to\infty} G(z_n, z_n, gu) = \lim_{n\to\infty} G(gx_n, gx_n, gu) = \lim_{n\to\infty} G(fx_n, fx_n, gu) = 0. \tag{2.39}$$

Now, we will prove that u is a coincidence point of f and g.

We know that $\{gx_n\}$ is a nondecreasing sequence in X. Regularity of X yields that $gx_n \preceq z = gu$. So, from (2.2) we have

$$\begin{aligned}
\psi(G(z_{n+1}, z_{n+1}, fu)) &= \psi(G(fx_n, fx_n, fu)) \\
&\leq \psi\left(\frac{G(gx_n, fx_n, fx_n) + G(gx_n, fu, fu) + G(gu, fx_n, fx_n)}{3}\right) \\
&\quad - \varphi(G(gx_n, fx_n, fx_n), G(gx_n, fu, fu), G(gu, fx_n, fx_n)) \\
&= \psi\left(\frac{G(z_n, z_{n+1}, z_{n+1}) + G(z_n, fu, fu) + G(gu, z_{n+1}, z_{n+1})}{3}\right) \\
&\quad - \varphi(G(z_n, z_{n+1}, z_{n+1}) + G(z_n, fu, fu) + G(gu, z_{n+1}, z_{n+1})).
\end{aligned} \tag{2.40}$$

Letting $n \to \infty$ in (2.40), from the continuity of ψ and φ, we get

$$\psi(G(z, z, fu)) \leq \psi\left(\frac{G(z, fu, fu)}{3}\right) - \varphi(0, G(z, fu, fu), 0). \tag{2.41}$$

As $G(z, fu, fu) \leq 2G(z, z, fu)$, we have

$$\psi(G(z, z, fu)) \leq \psi\left(\frac{2G(z, z, fu)}{3}\right) - \varphi(0, G(z, fu, \mathrm{f}u), 0). \tag{2.42}$$

Hence, $\varphi(0, G(z, fu, fu), 0) \leq \psi(2G(z, z, fu)/3) - \psi(G(z, z, fu)) \leq 0$. So, $G(z, fu, fu) = 0$ and hence, $gu = z = fu$. This means that g and f have a coincidence point. □

Taking $g = I_X$ (the identity mapping on X) and $\psi = I_{[0,\infty)}$ in the above theorems, we obtain the following fixed point result.

Corollary 2.3. *Let $(X, \preceq, G)$ be a partially ordered complete G-metric space. Let $f : X \to X$ be a mapping such that $fx \preceq f(fx)$, for all $x \in X$ and*

$$\begin{aligned}
G(fx, fy, fz) \leq{}& \frac{G(x, fy, fy) + G(y, fz, fz) + G(z, fx, fx)}{3} \\
&- \varphi(G(x, fy, fy), G(y, fz, fz), G(z, fx, fx)),
\end{aligned} \tag{2.43}$$

for every $x, y, z \in X$ such that $x \preceq y \preceq z$, where $\psi : [0, \infty) \to [0, \infty)$ is an altering distance function and $\varphi \in \Phi$. Then, f has a fixed point in X provided that one of the following two conditions is satisfied:

(a) *f is continuous, or,*

(b) *X is regular.*

Taking $\varphi(x,y,z) = (1/3-\alpha)(x+y+z)$, where $\alpha \in [0,1/3)$, in the above corollary, we obtain the following result.

Corollary 2.4. *Let $(X,\preceq,G)$ be a partially ordered complete G-metric space. Let $f : X \to X$ be a mapping such that $fx \preceq f(fx)$, for all $x \in X$ and*

$$G(fx,fy,fz) \le \alpha\big(G(x,fx,fx) + G(y,fy,fy) + G(z,fz,fz)\big), \tag{2.44}$$

for every $x,y,z \in X$ such that $x \preceq y \preceq z$, where $\alpha \in [0,1/3)$. Then, f has a fixed point in X if one of the following two conditions is satisfied:

(a) *f is continuous, or,*

(b) *X is regular.*

Theorem 2.5. *Under the hypotheses of Theorem 2.1, f and g have a common fixed point in X if g is a nondecreasing dominating map.*

Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Proof. Following the proof of the Theorem 2.1 we obtain that the sequence $\{z_n\}$ is G-convergent to z and $fz = gz$. Since f and g are weakly compatible (since the pair (f,g) is compatible), we have $fgz = gfz$. Let $w = gz = fz$. Therefore, we have

$$fw = gw. \tag{2.45}$$

As g is a nondecreasing dominating map,

$$z \preceq gz \preceq ggz = gw. \tag{2.46}$$

If $z = w$, then z is a common fixed point. If $z \ne w$, then, since from (2.46) $gz \preceq gw$, from (2.2) we have

$$\begin{aligned}
\psi(G(fz,fz,fw)) &\le \psi\left(\frac{G(gz,fz,fz)+G(gz,fw,fw)+G(gw,fz,fz)}{3}\right)\\
&\quad - \varphi(G(gz,fz,fz),G(gz,fw,fw),G(gw,fz,fz))\\
&\le \psi\left(\frac{G(fz,fz,fz)+G(fz,fw,fw)+G(fw,fz,fz)}{3}\right)\\
&\quad - \varphi(G(fz,fz,fz),G(fz,fw,fw),G(fw,fz,fz))\\
&\le \psi\left(\frac{2G(fz,fz,fw)+G(fz,fz,fw)}{3}\right)\\
&\quad - \varphi(0,G(fz,fw,fw),G(fw,fz,fz)).
\end{aligned} \tag{2.47}$$

Therefore, $\varphi(0, G(fz, fw, fw), G(fw, fz, fz)) = 0$. So, $fz = fw$. Now, since $w = gz = fz$ and $fw = gw$, we have $w = gw = fw$. This completes the proof.

Suppose that the set of common fixed points of f and g is well ordered. We claim that common fixed point of f and g is unique. Assume on contrary that, $fu = gu = u$ and $fv = gv = v$, and $u \neq v$. Without any loss of generality, we may assume that $gu = u \preceq v = gv$. Using (2.2), we obtain

$$\begin{aligned}
\psi(G(u,u,v)) &= \psi(G(fu,fu,fv)) \\
&\leq \psi\left(\frac{G(gu,fu,fu) + G(gu,fv,fv) + G(gv,fu,fu)}{3}\right) \\
&\quad - \varphi(G(gu,fu,fu), G(gu,fv,fv), G(gv,fu,fu)) \\
&\leq \psi\left(\frac{2G(v,u,u) + G(v,u,u)}{3}\right) \\
&\quad - \varphi(0, G(u,v,v), G(v,u,u)).
\end{aligned} \tag{2.48}$$

Therefore, $u = v$, a contradiction. Conversely, if f and g have only one common fixed point then, clearly, the set of common fixed points of f and g is well ordered. □

Theorem 2.6. *Under the hypotheses of Theorem 2.2, f and g have a common fixed point in X provided that f and g are weakly compatible and g is a nondecreasing dominating map.*

Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Proof. The proof is done as in Theorem 2.5. □

Following arguments similar to those given in the proof of Theorems 2.1 and 2.2, we have the following results for a generalized weakly G-contractive mapping of type B.

Theorem 2.7. *Let $(X, \preceq, G)$ be a partially ordered complete G-metric space. Let $f, g : X \to X$ be two mappings such that $f(X) \subseteq g(X) f$ is weakly increasing with respect to g and*

$$\begin{aligned}
\psi(G(fx,fy,fz)) \leq{} & \psi\left(\frac{G(gx,gx,fy) + G(gy,gy,fz) + G(gz,gz,fx)}{3}\right) \\
& - \varphi(G(gx,gx,fy), G(gy,gy,fz), G(gz,gz,fx)),
\end{aligned} \tag{2.49}$$

for every $x, y, z \in X$ such that $gx \preceq gy \preceq gz$, where $\psi : [0,\infty) \to [0,\infty)$ is an altering distance function and $\varphi \in \Phi$. Then f and g have a coincidence point in X provided that f and g are continuous and the pair (f, g) is compatible.

Moreover, f and g have a common fixed point in X if g is a nondecreasing dominating map.

Also, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Theorem 2.8. *Let $(X, \preceq, G)$ be a partially ordered G-metric space. Let $f, g : X \to X$ be two mappings such that $f(X) \subseteq g(X) f$ is weakly increasing with respect to g and*

$$\psi(G(fx, fy, fz)) \leq \psi\left(\frac{G(gx, gx, fy) + G(gy, gy, fz) + G(gz, gz, fx)}{3}\right) - \varphi(G(gx, gx, fy), G(gy, gy, fz), G(gz, gz, fx)), \tag{2.50}$$

for every $x, y, z \in X$ such that $gx \preceq gy \preceq gz$, where $\psi : [0, \infty) \to [0, \infty)$ is an altering distance function and $\varphi \in \Phi$. Then f and g have a coincidence point in X provided that X is regular and $g(X)$ is a G-complete subset of (X, G).

Moreover, f and g have a common fixed point in X if f and g are weakly compatible and g is a nondecreasing dominating map.

Also, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

The following corollary is an immediate consequence of the above theorems.

Corollary 2.9. *Let $(X, \preceq, G)$ be a partially ordered complete G-metric space. Let $f : X \to X$ be a mapping such that $fx \preceq f(fx)$, for all $x \in X$ and*

$$G(fx, fy, fz) \leq \frac{G(x, x, fy) + G(y, y, fz) + G(z, z, fx)}{3} - \varphi(G(x, x, fy), G(y, y, fz), G(z, z, fx)), \tag{2.51}$$

for every $x, y, z \in X$ such that $x \preceq y \preceq z$, where $\psi : [0, \infty) \to [0, \infty)$ is an altering distance function and $\varphi \in \Phi$. Then f has a fixed point in X provided that one of the following two conditions is satisfied:

(a) *f is continuous, or,*

(b) *X is regular.*

Example 2.10. Let $X = [0, \infty)$ be endowed with the usual order in $\mathbb{R}$ and G on X be given as

$$G(x, y, z) = \max\{|x - y|, |y - z|, |z - x|\}. \tag{2.52}$$

Define $f, g : X \to X$ as

$$f(x) = 1, \qquad g(x) = \begin{cases} 2 - x^2, & \text{if } 0 \leq x \leq \sqrt{2} \\ 0, & \text{if } x > \sqrt{2}, \end{cases} \tag{2.53}$$

for all $x \in X$.

Define $\psi : [0, \infty) \to [0, \infty)$ by $\psi(t) = (1/4)t^2$ and $\varphi : [0, \infty)^3 \to [0, \infty)$ by $\varphi(s, t, u) = (1/100)(s + t + u)^2$.

Let $0 \le x \le y \le z \le \sqrt{2}$. Now, we have

$$\begin{aligned}
\psi(G(fx,fy,fz)) = 0 &\le \frac{1}{4}\left(\frac{|x^2-1|+|y^2-1|+|z^2-1|}{3}\right)^2 \\
&\quad - \frac{1}{100}\left(|x^2-1|+|y^2-1|+|z^2-1|\right)^2 \\
&\le \frac{1}{4}\left(\frac{3-x^2-y^2-z^2}{3}\right)^2 - \frac{1}{100}\left(3-x^2-y^2-z^2\right)^2 \\
&= \psi\left(\frac{1}{3}(G(gx,fx,fx)+G(gy,fy,fy)+G(gz,fz,fz))\right) \\
&\quad - \varphi(G(gx,fx,fx),G(gy,fy,fy),G(gz,fz,fz)).
\end{aligned} \tag{2.54}$$

There are other 3 cases as follows:

(1) $0 \le x \le y \le 1$ and $\sqrt{2} < z$.

(2) $0 \le x \le \sqrt{2}$ and $\sqrt{2} < y \le z$.

(3) $\sqrt{2} < x \le y \le z$.

By a careful calculation for the remained cases above, we see that all the conditions of Theorems 2.1 and 2.5 are satisfied. Moreover, (1) is the unique common fixed point of f and g.

Denote by Λ the set of all functions $\mu : [0,+\infty) \to [0,+\infty)$ verifying the following conditions:

(I) μ is a positive Lebesgue integrable mapping on each compact subset of $[0,+\infty)$.

(II) for all $\varepsilon > 0$, $\int_0^{\varepsilon} \mu(t)dt > 0$.

Other consequences of the main theorems are the following results for mappings satisfying a contraction of integral type.

Corollary 2.11. *Replace the contractive condition* (2.2) *of Theorem 2.1 by the following condition. There exists a* $\mu \in \Lambda$ *such that*

$$\begin{aligned}
\int_0^{\psi(G(fx,fy,fz))} \mu(t)dt &\le \int_0^{\psi((G(gx,fy,fy)+G(gy,fz,fz)+G(gz,fx,fx))/3)} \mu(t)dt \\
&\quad - \int_0^{\varphi(G(gx,fy,fy),G(gy,fz,fz),G(gz,fx,fx))} \mu(t)dt.
\end{aligned} \tag{2.55}$$

Then, f and g have a coincidence point, if the other conditions of Theorem 2.1 are satisfied.

Proof. Consider the function $\Gamma(x) = \int_0^x \mu(t)dt$. Then, (2.55) becomes

$$\begin{aligned}\Gamma(\psi(G(fx,fy,fz))) \le{}& \Gamma\left(\psi\left(\frac{G(gx,fy,fy)+G(gy,fz,fz)+G(gz,fx,fx)}{3}\right)\right)\\&-\Gamma(\varphi(G(gx,fy,fy),G(gy,fz,fz),G(gz,fx,fx))).\end{aligned}\tag{2.56}$$

Taking $\psi_1 = \Gamma\circ\psi$ and $\varphi_1 = \Gamma\circ\varphi$ and applying Theorem 2.1, we obtain the proof (it is easy to verify that ψ_1 is an altering distance function and $\varphi_1 \in \Phi$). □

Similar to [21], let $N \in N^*$ be fixed. Let $\{\mu_i\}_{1\le i\le N}$ be a family of N functions which belong to Λ. For all $t \ge 0$, we define

$$\begin{aligned}I_1(t) &= \int_0^t \mu_1(s)ds,\\ I_2(t) &= \int_0^{I_1 t} \mu_2(s)ds = \int_0^{\int_0^t \mu_1(s)ds} \mu_2(s)ds,\\ I_3(t) &= \int_0^{I_2 t} \mu_3(s)ds = \int_0^{\int_0^{\int_0^t \mu_1(s)ds} \mu_2(s)ds} \mu_3(s)ds,\\ &\vdots\\ I_N(t) &= \int_0^{I_{(N-1)}t} \mu_N(s)ds.\end{aligned}\tag{2.57}$$

We have the following result.

Corollary 2.12. *Replace the inequality* (2.2) *of Theorem 2.1 by the following condition:*

$$\begin{aligned}I_N(\psi(G(fx,fy,fz))) \le{}& I_N\left(\psi\left(\frac{G(gx,fy,fy)+G(gy,fz,fz)+G(gz,fx,fx)}{3}\right)\right)\\&-I_N(\varphi(G(gx,fy,fy),G(gy,fz,fz),G(gz,fx,fx))).\end{aligned}\tag{2.58}$$

Then, f and g have a coincidence point if the other conditions of Theorem 2.1 are satisfied.

Proof. Consider $\widehat{\Psi} = I_N\circ\psi$ and $\widehat{\Phi} = I_N\circ\varphi$. □

3. Periodic Point Results

Let $F(f) = \{x \in X : fx = x\}$, the fixed point set of f.

Clearly, a fixed point of f is also a fixed point of f^n for every $n \in \mathbb{N}$; that is, $F(f) \subset F(f^n)$. However, the converse is false. For example, the mapping $f : \mathbb{N} \to \mathbb{N}$, defined by $fx = 1/2 - x$ has the unique fixed point $1/4$, but every $x \in \mathbb{N}$ is a fixed point of f^2.

If $F(f) = F(f^n)$ for every $n \in \mathbb{N}$, then f is said to have property P. For more details, we refer the reader to [6, 26–28] and the references mentioned therein.

Theorem 3.1. *Let X and f be as in Corollary 2.3. If f is a dominating map on X, then f has property P.*

Proof. From Corollary 2.3, $F(f) \neq \emptyset$. Let $u \in F(f^n)$ for some $n > 1$. We will show that $u = fu$. Since f is dominating on X, we have $u \preceq fu$, which implies that $f^{n-1}u \preceq f^n u$, as f is nondecreasing. Using (2.2), we obtain that

$$\begin{aligned}
G(u, fu, fu) &= G\left(f^n u, f^{n+1}u, f^{n+1}u\right) \\
&= G\left(f f^{n-1}u, f f^n u, f f^n u\right) \\
&\leq \frac{1}{3}\left(G\left(f^{n-1}u, f^{n+1}u, f^{n+1}u\right) + G\left(f^n u, f^{n+1}u, f^{n+1}u\right) + G(f^n u, f^n u, f^n u)\right) \\
&\quad - \varphi\left(G\left(f^{n-1}u, f^{n+1}u, f^{n+1}u\right), G\left(f^n u, f^{n+1}u, f^{n+1}u\right), G(f^n u, f^n u, f^n u)\right) \\
&\leq \frac{1}{3}\left(G\left(f^{n-1}u, f^n u, f^n u\right) + 2G\left(f^n u, f^{n+1}u, f^{n+1}u\right) + 0\right) \\
&\quad - \varphi\left(G\left(f^{n-1}u, f^{n+1}u, f^{n+1}u\right), G\left(f^n u, f^{n+1}u, f^{n+1}u\right), 0\right),
\end{aligned} \tag{3.1}$$

that is,

$$\begin{aligned}
G(u, fu, fu) &= G\left(f^n u, f^{n+1}u, f^{n+1}u\right) \\
&\leq G\left(f^{n-1}u, f^n u, f^n u\right) \\
&\quad - 3\varphi\left(G\left(f^{n-1}u, f^{n+1}u, f^{n+1}u\right), G\left(f^n u, f^{n+1}u, f^{n+1}u\right), 0\right).
\end{aligned} \tag{3.2}$$

Repeating the above process, we get

$$\begin{aligned}
&G\left(f^{n-(i)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\right) \\
&\quad \leq G\left(f^{n-(i+1)}u, f^{n-(i)}u, f^{n-(i)}u\right) \\
&\qquad - 3\varphi\left(G\left(f^{n-(i+1)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\right) G\left(f^{n-(i)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\right), 0\right).
\end{aligned} \tag{3.3}$$

From the above inequalities, we have

$$\begin{aligned}
G(u, fu, fu) &\leq G(u, fu, fu) \\
&\quad - 3\sum_{i=0}^{n-1} \varphi\left(G\left(f^{n-(i+1)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\right), G\left(f^{n-(i)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\right), 0\right).
\end{aligned} \tag{3.4}$$

Therefore,

$$\sum_{i=0}^{n-1}\varphi\Big(G\Big(f^{n-(i+1)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\Big), G\Big(f^{n-(i)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\Big), 0\Big) = 0, \tag{3.5}$$

which from our assumptions about φ implies that

$$G\Big(f^{n-(i+1)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\Big) = G\Big(f^{n-(i)}u, f^{n-(i-1)}u, f^{n-(i-1)}u\Big) = 0 \tag{3.6}$$

for all $0 \leq i \leq n-1$. Now, taking $i = n-1$, we have $u = fu$. □

Analogously, we have the following theorem.

Theorem 3.2. *Let X and f be as in Corollary 2.12. If f is a dominating map on X, then f has property P.*

References

[1] Z. Mustafa and B. Sims, "A new approach to generalized metric spaces," *Journal of Nonlinear and Convex Analysis*, vol. 7, no. 2, pp. 289–297, 2006.

[2] M. Abbas, T. Nazir, and D. Dorić, "Common fixed point of mappings satisfying (E, A) property in generalized metric spaces," *Applied Mathematics and Computation*, vol. 218, no. 14, pp. 7665–7670, 2012.

[3] M. Abbas, T. Nazir, and S. Radenović, "Common fixed point of generalized weakly contractive maps in partially ordered G-metric spaces," *Applied Mathematics and Computation*, vol. 218, no. 18, pp. 9383–9395, 2012.

[4] M. Abbas, T. Nazir, and S. Radenović, "Some periodic point results in generalized metric spaces," *Applied Mathematics and Computation*, vol. 217, no. 8, pp. 4094–4099, 2010.

[5] H. Aydi, W. Shatanawi, and C. Vetro, "On generalized weak G-contraction mapping in G-metric spaces," *Computers & Mathematics with Applications*, vol. 62, no. 11, pp. 4223–4229, 2011.

[6] M. Khandaqji, Sh. Al-Sharif, and M. Al-Khaleel, "Properpty P and some fixed point results on (ψ, φ)-weakly contractive G-metric spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 2012, Article ID 675094, 11 pages, 2012.

[7] Z. Mustafa, M. Khandagji, and W. Shatanawi, "Fixed point results on complete G-metric spaces," *Studia Scientiarum Mathematicarum Hungarica*, vol. 48, no. 3, pp. 304–319, 2011.

[8] Z. Mustafa and H. Obiedat, "A fixed point theorem of Reich in G-metric spaces," *CUBO: A Mathematical Journal*, vol. 12, no. 1, pp. 83–93, 2010.

[9] Z. Mustafa, H. Obiedat, and F. Awawdeh, "Some fixed point theorem for mapping on complete G-metric spaces," *Fixed Point Theory and Applications*, vol. 2008, Article ID 189870, 12 pages, 2008.

[10] Z. Mustafa, W. Shatanawi, and M. Bataineh, "Existence of fixed point results in G-metric spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 2009, Article ID 283028, 10 pages, 2009.

[11] Z. Mustafa and B. Sims, "Fixed point theorems for contractive mappings in complete G-metric spaces," *Fixed Point Theory and Applications*, vol. 2009, Article ID 917175, 10 pages, 2009.

[12] W. Shatanawi, "Fixed point theory for contractive mappings satisfying Φ-maps in G-metric spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 181650, 9 pages, 2010.

[13] W. Shatanawi, "Some fixed point theorems in ordered G-metric spaces and applications," *Abstract and Applied Analysis*, vol. 2011, Article ID 126205, 11 pages, 2011.

[14] N. Tahat, H. Aydi, E. Karapınar, and W. Shatanawi, "Common fixed point for single-valued and multi-valued maps satisfying a generalized contraction in G-metric spaces," *Fixed Point Theory and Applications*, vol. 2012, article 48, 2012.

[15] B. S. Choudhury and P. Maity, "Coupled fixed point results in generalized metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 1-2, pp. 73–79, 2011.

[16] M. S. Khan, M. Swaleh, and S. Sessa, "Fixed point theorems by altering distances between the points," *Bulletin of the Australian Mathematical Society*, vol. 30, no. 1, pp. 1–9, 1984.

[17] B. S. Choudhury, "Unique fixed point theorem for weak C-contractive mappings," *Kathmandu University Journal of Science, Engineering and Technology*, vol. 5, no. 1, pp. 6–13, 2009.

[18] J. Harjani, B. Lopez, and K. Sadarangani, "Fixed point theorems for weakly C-contractive mappings in ordered metric spaces," *Computers & Mathematics with Applications*, vol. 61, no. 4, pp. 790–796, 2011.

[19] W. Shatanawi, "Fixed point theorems for nonlinear weakly C-contractive mappings in metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 11-12, pp. 2816–2826, 2011.

[20] M. Abbas, T. Nazir, and S. Radenović, "Common fixed points of four maps in partially ordered metric spaces," *Applied Mathematics Letters*, vol. 24, no. 9, pp. 1520–1526, 2011.

[21] H. K. Nashine and B. Samet, "Fixed point results for mappings satisfying (ψ,φ)-weakly contractive condition in partially ordered metric spaces," *Nonlinear Analysis*, vol. 74, no. 6, pp. 2201–2209, 2011.

[22] G. Jungck, "Compatible mappings and common fixed points," *International Journal of Mathematics and Mathematical Sciences*, vol. 9, no. 4, pp. 771–779, 1986.

[23] I. Altun and H. Simsek, "Some fixed point theorems on ordered metric spaces and application," *Fixed Point Theory and Applications*, vol. 2010, Article ID 621469, 17 pages, 2010.

[24] W. Shatanawi and B. Samet, "On (ψ,φ)-weakly contractive condition in partially ordered metric spaces," *Computers & Mathematics with Applications*, vol. 62, no. 8, pp. 3204–3214, 2011.

[25] M. Kumar, "Compatible maps in G-metric spaces," *International Journal of Mathematical Analysis*, vol. 6, no. 29, pp. 1415–1421, 2012.

[26] M. Abbas, V. Parvaneh, and A. Razani, "Periodic point of T-Ciric generalized contraction mappings in ordered metric spaces," *Georgian Mathematical Journal*. In press.

[27] R. Chugh, T. Kadian, A. Rani, and B. E. Rhoades, "Property P in G-metric spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 401684, 12 pages, 2010.

[28] G. S. Jeong and B. E. Rhoades, "Maps for which $F(T) = F(T^n)$," *Fixed Point Theory and Applications*, vol. 6, pp. 87–131, 2005.