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Research Article

Design of PDC Controllers by Matrix Reversibility for Synchronization of *Yin* and *Yang* Chaotic Takagi-Sugeno Fuzzy Henon Maps

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This paper investigates the synchronization of *Yin* and *Yang* chaotic T-S fuzzy Henon maps via PDC controllers. Based on the Chinese philosophy, *Yin* is the decreasing, negative, historical, or feminine principle in nature, while *Yang* is the increasing, positive, contemporary, or masculine principle in nature. *Yin* and *Yang* are two fundamental opposites in Chinese philosophy. The Henon map is an invertible map; so the Henon maps with increasing and decreasing argument can be called the *Yang* and *Yin* Henon maps, respectively. Chaos synchronization of *Yin* and *Yang* T-S fuzzy Henon maps is achieved by PDC controllers. The design of PDC controllers is based on the linear invertible matrix theory. The T-S fuzzy model of *Yin* and *Yang* Henon maps and the design of PDC controllers are novel, and the simulation results show that the approach is effective.

1. Introduction

Chaotic behavior in nonlinear system is an interesting phenomenon, which is very sensitive in initial conditions. Chaos plays an important role in nonlinear science field [1–4]. In [5], the *Yin* chaos in continuous time nonlinear system is firstly proposed. In 1990, Pecora and Carroll proposed chaotic synchronization scheme and started a new research interest [6]. In many papers, chaos synchronization of two-dimensional Hênon map system is studied [7–11]. In 1985, Takagi and Sugeno [12] proposed T-S fuzzy model and has been used in complex nonlinear systems successfully [13]. The T-S fuzzy model of the nonlinear systems can exactly be represented as a fuzzy aggregation of some local linear systems. Generally, the implementation of synchronization [14] and control [15, 16] in T-S fuzzy model of chaotic systems uses the parallel distributed compensation (PDC) scheme.

The features of this paper are as follows.

(1) We develop the chaotic Henon map with decreasing argument, which is called *Yin* chaotic Henon map.

(2) *Yin* and *Yang* T-S fuzzy models of chaotic Henon maps are proposed.

(3) The nonlinear discrete time *Yin* Henon map is the inverse of *Yang* Henon map. Besides, the design of PDC controllers uses linear invertible matrix theory to achieve chaos synchronization of T-S fuzzy *Yin* and *Yang* Henon maps.

This paper is organized as follows. In Section 2, the two-dimensional invertible maps, *Yin* and *Yang* chaos are introduced. In Section 3, the *Yang* and *Yin* chaos of Henon maps are presented. Section 4, Takagi-Sugeno fuzzy model scheme is given. In Section 5, *Yang* and *Yin* T-S fuzzy models of the chaotic Henon maps are given. In Section 6, the design of PDC controllers for *Yin* and *Yang* T-S fuzzy synchronization of chaotic Henon maps is given by the linear invertible matrix theory. In Section 7, simulation results are given. In Section 8, conclusions are drawn.

2. Two-Dimensional Invertible Maps

Consider the general two-dimensional map form as follows:

$$\begin{aligned} x_1[n_1+1] &= f_1\{x_1[n_1], x_2[n_1]\}, \\ x_2[n_1+1] &= f_2\{x_1[n_1], x_2[n_1]\}, \end{aligned} \tag{2.1}$$

where $x_1[n_1+1]$, $x_2[n_1+1]$ are the functions of $x_1[n_1]$, $x_2[n_1]$, n_1 is the positive argument, that is, $n_1 = 0, 1, 2, 3, \ldots, n$. The map is invertible [17] if (2.1) can be solved uniquely for $x_1[n_1]$ and $x_2[n_1]$ as functions of $x_1[n_1+1]$ and $x_2[n_1+1]$. The inversed map system can be written from (2.1):

$$\begin{aligned} \hat{x}_1[n_2-1] &= g_1\{\hat{x}_1[n_2], \hat{x}_2[n_2]\}, \\ \hat{x}_2[n_2-1] &= g_2\{\hat{x}_1[n_2], \hat{x}_2[n_2]\}, \end{aligned} \tag{2.2}$$

where n_2 is the negative argument, that is, $n_2 = 0, -1, -2, -3, \ldots, -n$. The chaos obtained for negative argument is called *Yin* chaos, while that obtained for positive argument is called *Yang* chaos.

3. The *Yang* Chaos of Henon Map System and *Yin* Chaos of Inversed Henon Map System

Consider the Henon map system [18]:

$$\begin{aligned} x_1[n_1+1] &= -a_1 x_1^2[n_1] + x_2[n_1] + 1, \\ x_2[n_1+1] &= b_1 x_1[n_1], \end{aligned} \tag{3.1}$$

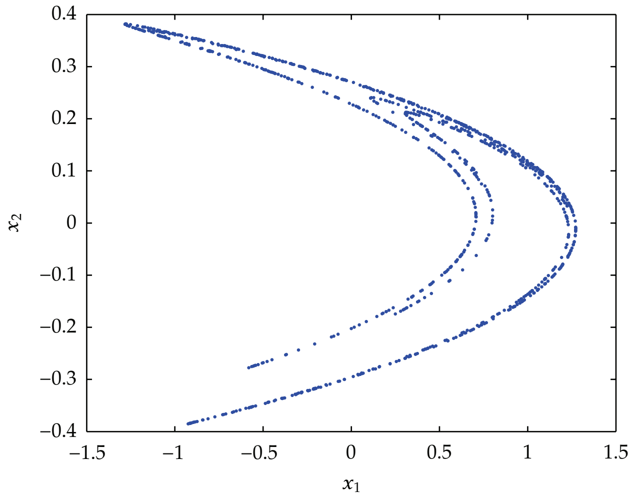

Figure 1: Phase portrait of the chaotic *Yang* Henon map system with $a_1 = 1.4, b_1 = 0.3$.

where $a_1 = 1.4, b_1 = 0.3$ and positive argument $n_1 = 0, 1, 2, 3, \ldots, n$. The *Yang* chaotic behaviors of Henon map system are quoted [19] by phase portraits in Figure 1.

The Henon map system is invertible if $b_1 \neq 0$. The inversed Henon map system can be described as follows:

$$\begin{aligned} \hat{x}_1[n_2 - 1] &= \frac{1}{b_1}\hat{x}_2[n_2], \\ \hat{x}_2[n_2 - 1] &= \hat{x}_1[n_2] + \frac{a_1}{b_1^2}\hat{x}_2^2[n_2] - 1. \end{aligned} \tag{3.2}$$

The parameters $1/b_1$, a_1/b_1^2 are replaced by b_2, a_2, respectively, where $b_2 = 0.3$, $a_2 = 1.4$ and negative argument $n_2 = -1, -2, -3, \ldots, -n$. The *Yin* chaotic behaviors of inversed Henon map system are shown in Figure 2.

Comparing Figure 1 and Figure 2, it is surprisingly noted that Figure 2 gives new information for famous Henon map system. Since 1976 [18], traditional studies of Henon system have only been devoted to its behaviors with positive argument. Now it is discovered that with negative argument or negative time, a new continent is waiting for us in the future study of either nonlinear map systems or nonlinear continuous systems.

4. Takagi-Sugeno Fuzzy Model

The T-S fuzzy model [12] can represent a nonlinear system by linear state space which is described by fuzzy IF-THEN rules.

Consider the following discrete time T-S fuzzy rules:

$$\begin{gathered} R^i: \text{ IF } p_1(n) \text{ is } M_{i1}, \ldots, \text{ and } p_q(n) \text{ is } M_{iq}, \\ \text{THEN } x(n+1) = A_i x(n) + c, \end{gathered} \tag{4.1}$$

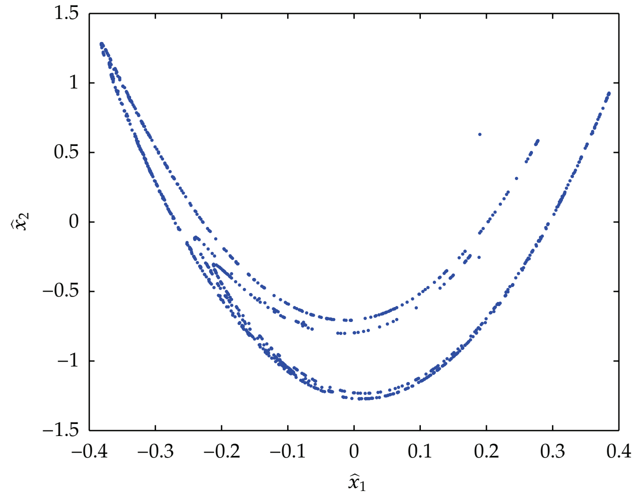

Figure 2: Phase portrait of the chaotic *Yin* Henon map system with $a_2 = 1.4, b_2 = 0.3$.

where $i = 1, 2, \ldots, r$ (r is the number of fuzzy rules), $x(n) \in R^j$ represents the state vector, $A_i \in R^{j\times j}$ are known matrix, $p_1(n), p_2(n), \ldots, p_q(n)$ are premise variables, M_{ih} is a fuzzy set ($h = 1, 2, \ldots, q$), and $c \in R^j$ is a constant vector. The final discrete time T-S fuzzy system is inferred as follows:

$$x(n+1) = \frac{\sum_{i=1}^{r} \omega_i(p(n))(A_i x(n) + c)}{\sum_{i=1}^{r} \omega_i(p(n))}, \tag{4.2}$$

where

$$\omega_i(p(n)) = \prod_{h=1}^{q} M_{ih}(p_h(n)), \tag{4.3}$$

in which $M_{ih}(p_h(n))$ is the degree of membership of $p_h(n)$ in M_{ih}. The following conditions must be satisfied:

$$\begin{aligned} &\sum_{i=1}^{r} \omega_i(p(n)) > 0, \\ &\omega_i(p(n)) \geq 0, \end{aligned} \qquad i = 1, 2, \ldots, r. \tag{4.4}$$

let $\mu_i(p(n)) = \omega_i(p(n)) / \sum_{i=1}^{r} \omega_i(p(n))$, (4.2) can be rewritten as

$$x(n+1) = \sum_{i=1}^{r} \mu_i(p(n))(A_i x(n) + c). \tag{4.5}$$

Note that

$$\begin{aligned}&\sum_{i=1}^{r}\mu_i(p(n)) = 1, \qquad i = 1,2,\ldots,r.\\&\mu_i(p(n)) \geq 0.\end{aligned} \tag{4.6}$$

5. *Yang* and *Yin* T-S Fuzzy Models of the Chaotic Henon Maps

Figures 1 and 2 show that the ranges of the states $x_1(n)$ and $\hat{x}_2(n)$ for *Yang* and Henon map are from -1.4 to 1.4 ($x_1(n), \hat{x}_2 \in [-1.4, 1.4]$), respectively. The *Yang* and *Yin* T-S fuzzy models of chaotic Henon map systems can be obtained as follows.

Yang T-S fuzzy model of chaotic Henon map system:

$$\begin{aligned}&R^1 : \text{IF } x_1(n_1) \text{ is } M_{11}, \text{ THEN } x(n_1+1) = A_1 x(n_1) + c,\\&R^2 : \text{IF } x_1(n_1) \text{ is } M_{21}, \text{ THEN } x(n_1+1) = A_2 x(n_1) + c.\end{aligned} \tag{5.1}$$

Yin T-S fuzzy model of chaotic Henon map system:

$$\begin{aligned}&R^1 : \text{IF } \hat{x}_2(n_2) \text{ is } \widehat{M}_{11}, \text{ THEN } \hat{x}(n_2-1) = \widehat{A}_1 \hat{x}(n_2) + \hat{c},\\&R^2 : \text{IF } \hat{x}_2(n_2) \text{ is } \widehat{M}_{21}, \text{ THEN } \hat{x}(n_2-1) = \widehat{A}_2 \hat{x}(n_2) + \hat{c}.\end{aligned} \tag{5.2}$$

In (5.1), $x(n_1) = (x_1(n_1), x_2(n_1))^T$, $n_1 = 0,1,2,3,\ldots,n$, $A_1 = \begin{bmatrix} -a_1(1.4) & 1 \\ b_1 & 0 \end{bmatrix}$, and $A_2 = \begin{bmatrix} -a_1(-1.4) & 1 \\ b_1 & 0 \end{bmatrix}$, $c = (1,0)^T$.

The membership functions are chosen as follows:

$$\begin{aligned}M_{11}(x_1(n_1)) &= \frac{1}{2}\left(1 + \frac{x_1(n_1)}{1.4}\right),\\M_{21}(x_1(n_1)) &= \frac{1}{2}\left(1 - \frac{x_1(n_1)}{1.4}\right).\end{aligned} \tag{5.3}$$

In (5.2), $\hat{x}(n_2) = (\hat{x}_1(n_2), \hat{x}_2(n_2))^T$, $n_2 = 0,-1,-2,-3,\ldots,-n$, $\widehat{A}_1 = \begin{bmatrix} 0 & b_2 \\ 1 & a_2(1.4) \end{bmatrix}$, $\widehat{A}_2 = \begin{bmatrix} 0 & b_2 \\ 1 & a_2(-1.4) \end{bmatrix}$, and $\hat{c} = (0,-1)^T$.

The membership functions are chosen as follows:

$$\begin{aligned}\widehat{M}_{11}(\hat{x}_2(n_2)) &= \frac{1}{2}\left(1 + \frac{\hat{x}_2(n_2)}{1.4}\right)\\\widehat{M}_{21}(\hat{x}_2(n_2)) &= \frac{1}{2}\left(1 - \frac{\hat{x}_2(n_2)}{1.4}\right).\end{aligned} \tag{5.4}$$

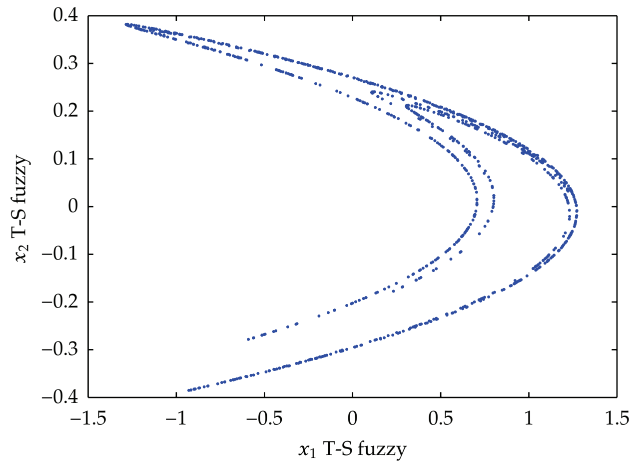

Figure 3: Phase portrait of *Yang* T-S fuzzy model of Henon map system with $a_1 = 1.4, b_1 = 0.3$.

Then, applying the product-inference rule, singleton fuzzifier, and the center of gravity defuzzifier to the above fuzzy rule base ((5.1) and (5.2)), the overall *Yang* and *Yin* fuzzy chaotic maps can be shown in (5.5) and (5.6), respectively

$$x(n_1+1) = \sum_{i=1}^{2} \mu_i(x_1(n_1))(A_i x(n_1) + c), \tag{5.5}$$

$$\hat{x}(n_2+1) = \sum_{i=1}^{2} \mu_i(\hat{x}_2(n_2))(A_i \hat{x}(n_2) + \hat{c}), \tag{5.6}$$

where

$$\begin{aligned} \mu_i(x_1(n_1)) &= \frac{M_{i1}(x_1(n_1))}{\sum_{i=1}^{2}(M_{i1}(x_1(n_1)))}, \\ \mu_i(\hat{x}_2(n_2)) &= \frac{M_{i1}(\hat{x}_2(n_2))}{\sum_{i=1}^{2}(M_{i1}(\hat{x}_2(n_2)))}. \end{aligned} \tag{5.7}$$

The *Yang* T-S fuzzy model of chaotic Henon map system is given in Figure 3. It is clear to see that the derived *Yang* T-S fuzzy model is equivalent to the original chaotic map in Figure 1. Otherwise, the *Yin* T-S fuzzy model of chaotic Henon map system is given in Figure 4 and is equivalent to the original chaotic map in Figure 2.

6. Controller Design for Chaos Synchronization of *Yin* and *Yang* T-S Fuzzy Model of Henon Maps

Let *Yang* T-S fuzzy chaotic map (5.5) be as the drive system and the response system as follows:

$$y(n+1) = \sum_{i=1}^{2} \mu_i(y_1(n))(A_i y(n) + c) + u(n). \tag{6.1}$$

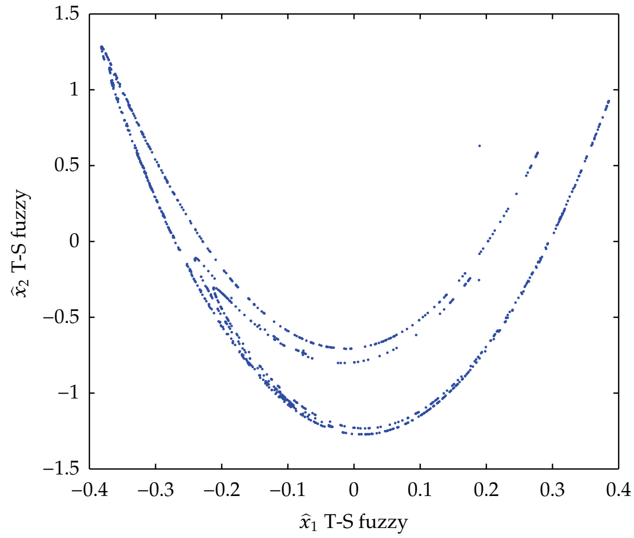

Figure 4: Phase portrait of *Yin* T-S fuzzy model of Henon map system with $a_2 = 1.4, b_2 = 0.3$.

The target for the design of a fuzzy controller $u(n)$ is synchronized the two discrete time chaotic maps by the PDC technique as follows:

$$u(n) = \sum_{i=1}^{2} \mu_i(x_1(n))K_i x(n) - \sum_{i=1}^{2} \mu_i(y_1(n))K_i y(n). \tag{6.2}$$

Define the error dynamics:

$$e(n) = x(n) - y(n). \tag{6.3}$$

From (6.1) and (6.2), the closed loop synchronization error dynamics (6.3) is arranged as

$$e(n+1) = \sum_{i=1}^{2} \mu_i(x_1(n))(A_i + K_i)x(n) - \sum_{i=1}^{2} \mu_i(y_1(n))(A_i + K_i)y(n). \tag{6.4}$$

According to stability theory for discrete linear system, it is known that a matrix H if and only if the eigenvalues of matrix H are less than 1 in absolute value [20], the error dynamics (6.3) is asymptotically stable, in other words, the chaos synchronization is achieved. Let a Schur stable matrix H_1 for *Yang* controller design satisfy the following equation:

$$A_1 + K_1 = A_2 - K_2 = H_1. \tag{6.5}$$

Since the system is the T-S fuzzy model of invertible map and the determinants of K_1, K_2 are not equal to zero, the *Yin* fuzzy controllers meet the following equation:

$$\widehat{K}_1 = \widehat{K}_1^{-1}, \qquad \widehat{K}_2 = \widehat{K}_2^{-1}. \tag{6.6}$$

So the Schur matrix H_2 for *Yin* controller design satisfy the following equation:

$$A_1^{-1} + K_1^{-1} = A_2^{-1} - K_2^{-1} = H_2. \tag{6.7}$$

We need to find the K_1, K_2, $\widehat{K}_1$, $\widehat{K}_2$ to meet (6.5) and (6.7), that is, if all the eigenvalues of H_1, H_2 are less than 1 in absolute value, the *Yang* and *Yin* T-S fuzzy chaotic Henon maps are synchronized individually. Because (6.7) is not the invertible of (6.5) (the invertible of (6.5) is $(A_1 + K_1)^{-1} = (A_2 - K_2)^{-1} = (H_1)^{-1}$), we can find H_1 and H_2 to meet Schur matrix.

7. Simulation Results

In order to apply the proposed *Yin-Yang* chaos synchronization scheme by PDC technique, the T-S fuzzy model is used. *Yang* T-S fuzzy model of Henon map system as drive system is described in (5.1), (5.3), (5.5), and (5.7). *Yang* T-S fuzzy model of Henon map system as response system is described by (6.1) and (6.2):

$$y(n_1 + 1) = \sum_{i=1}^{2} \mu_i(y_1(n_1))\left(A_i y(n_1) + c\right) + u(n_1), \tag{7.1}$$

where

$$\begin{gathered} A_1 = \begin{bmatrix} -a_1(1.4) & 1 \\ b_1 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -a_1(-1.4) & 1 \\ b_1 & 0 \end{bmatrix}, \qquad c = [1, 0]^T, \\ u(n) = \sum_{i=1}^{2} \mu_i(x_1(n_1)) K_i x(n_1) - \sum_{i=1}^{2} \mu_i(y_1(n_1)) K_i y(n_1). \end{gathered} \tag{7.2}$$

According to the proposed *Yin-Yang* chaos synchronization by PDC technique, the stable matrix H_1 is chosen as $H_1 = \left[\begin{smallmatrix} -0.96 & 0.5 \\ -0.7 & 0 \end{smallmatrix}\right]$, where the two eigenvalues of H_1 are $-0.48, -0.48$. The *Yang* fuzzy controller can be obtained by (6.5):

$$\begin{gathered} K_1 = \begin{bmatrix} 1 & -0.5 \\ -1 & 0 \end{bmatrix}, \\ K_2 = \begin{bmatrix} 2.92 & 0.5 \\ 1 & 0 \end{bmatrix}. \end{gathered} \tag{7.3}$$

Yang T-S fuzzy model of chaotic Henon maps as drive and response system is simulated with initial conditions $[\, x_1(0) \;\; x_2(0) \,]^T = [\, 0.63 \;\; 0.19 \,]^T$, $[\, y_1(0) \;\; y_2(0) \,]^T = [\, 0.4 \;\; 0.1 \,]^T$.

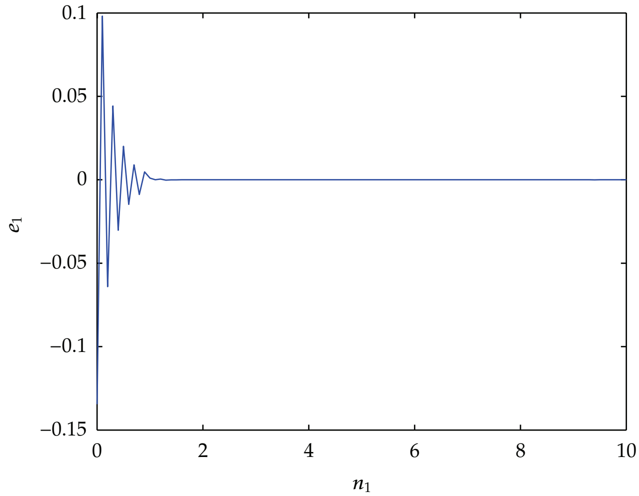

Figure 5: Error dynamics e_1 for chaos synchronization of *Yang* T-S fuzzy model of Henon map system.

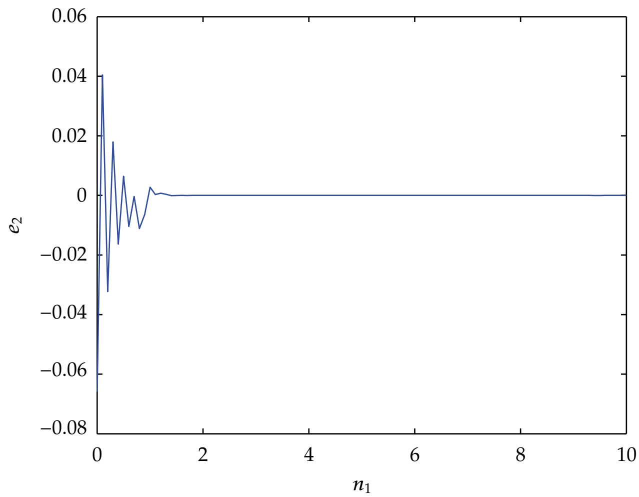

Figure 6: Error dynamics e_2 for chaos synchronization of *Yang* T-S fuzzy model of Henon map system.

Simulation results show that the error dynamics approach to be asymptotically stable in Figures 5 and 6. The *Yin* fuzzy controllers can be obtained by (6.6):

$$\widehat{K}_1 = K_1^{-1} = \begin{bmatrix} 0 & -1 \\ -2 & -2 \end{bmatrix},$$
$$\widehat{K}_2 = K_2^{-1} = \begin{bmatrix} 0 & 1 \\ 2 & -5.84 \end{bmatrix}. \tag{7.4}$$

From (6.7), we can obtain the Schur matrix $H_2 = \begin{bmatrix} 0 & -0.7 \\ -1 & -0.04 \end{bmatrix}$, where the initial conditions $\begin{bmatrix} \hat{x}_1(0) & \hat{x}_2(0) \end{bmatrix}^T = \begin{bmatrix} 0.19 & 0.63 \end{bmatrix}^T$, $\begin{bmatrix} \hat{y}_1(0) & \hat{y}_2(0) \end{bmatrix}^T = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix}^T$ and the two eigenvalues of H_2 are 0.817,

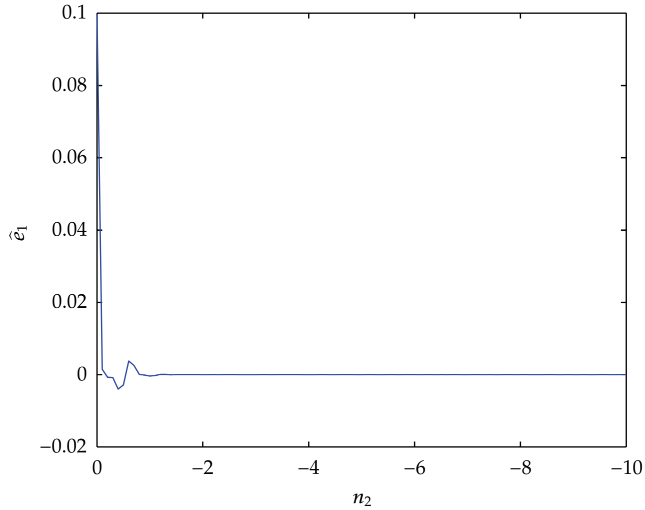

Figure 7: Error dynamics $\hat{e}_1$ for chaos synchronization of *Yin* T-S fuzzy model of Henon map system.

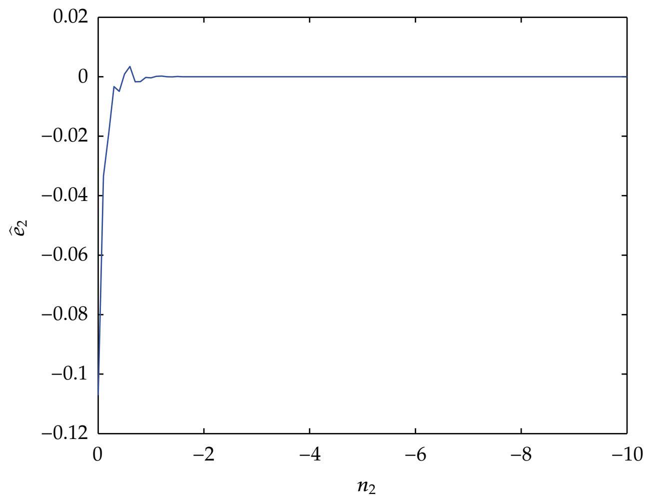

Figure 8: Error dynamics $\hat{e}_2$ for chaos synchronization of *Yin* T-S fuzzy model of Henon map system.

−0.857. Simulation results show that the error dynamics approach to be asymptotically stable in Figures 7 and 8. In other words, chaos synchronization is achieved.

8. Conclusions

Yin chaos of inversed Henon map system and the *Yin* T-S fuzzy model of chaotic Henon map system are proposed. The T-S fuzzy model of a chaotic system can exactly be represented as a fuzzy aggregation of some local linear systems. As a result, the conventional linear system theory can be applied. Since nonlinear discrete time Henon map is an invertible map, we develop the *Yang* and *Yin* T-S fuzzy model of Henon map system. The design of the *Yin* fuzzy controller is fleetly obtained by the inverse of *Yang* fuzzy controller, the concern is that the *Yin* chaotic system must be an inverse of the *Yang* chaotic system and meet the stability theory for discrete linear system. In secure communication, the chaotic system as transmitter and the inverse chaotic system as receiver are hard to achieve due to the nonlinearity; however the *Yin* and *Yang* T-S fuzzy models of Henon map system can overcome these difficulties.

Acknowledgments

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