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Research Article

Generalized Hyers-Ulam Stability of Generalized (N, K)-Derivations

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Let $3 \leq n$, and $3 \leq k \leq n$ be positive integers. Let A be an algebra and let X be an A-bimodule. A $\mathbb{C}$-linear mapping $d : A \to X$ is called a generalized (n, k)-derivation if there exists a $(k-1)$-derivation $\delta : A \to X$ such that $d(a_1 a_2 \cdots a_n) = \delta(a_1)a_2 \cdots a_n + a_1\delta(a_2)a_3 \cdots a_n + \cdots + a_1 a_2 \cdots a_{k-2}\delta(a_{k-1})a_k \cdots a_n + a_1 a_2 \cdots a_{k-1} d(a_k)a_{k+1} \cdots a_n + a_1 a_2 \cdots a_k d(a_{k+1})a_{k+2} \cdots a_n + a_1 a_2 \cdots a_{k+1} d(a_{k+2})a_{k+3} \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n)$ for all $a_1, a_2, \ldots, a_n \in A$. The main purpose of this paper is to prove the generalized Hyers-Ulam stability of the generalized (n, k)-derivations.

1. Introduction

It seems that the stability problem of functional equations introduced by Ulam [1]. *Let $(G_1, \cdot)$ be a group and let $(G_2, *)$ be a metric group with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta > 0$, such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x \cdot y), h(x) * h(y)) < \delta$, for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$, for all $x \in G_1$*? In other words, under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equations arises when one replaces the functional equation by an inequality which acts as a perturbation of the equation. In 1941, Hyers [2] gave the first affirmative answer to the question of Ulam for Banach spaces E and E'. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\|f(x+y) - f(x) - f(y)\| \leq \delta \tag{1.1}$$

for all $x, y \in E$, and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(x) - T(x)\| \leq \delta \tag{1.2}$$

for all $x \in E$. By the seminal paper of Th. M. Rassias [3] and work of Gadja [4], if one assumes that E and E' are real normed spaces with E' complete, $f : E \to E'$ is a mapping such that for each fixed $x \in E$ the mapping $t \rightarrowtail f(tx)$ is continuous in real t for each fixed x in E, and that there exists $\delta \geq 0$ and $p \neq 1$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \delta(\|x\|^p + \|y\|^p) \tag{1.3}$$

for all $x, y \in E$. Then there exists a unique linear map $T : E \to E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\delta\|x\|^p}{|2^p - 2|} \tag{1.4}$$

for all $x \in E$.

On the other hand J. M. Rassias [5] generalized the Hyers stability result by presenting a weaker condition controlled by a product of different powers of norms. If it is assumed that there exist constants $\Theta \geq 0$ and $p_1, p_2 \in \mathbb{R}$ such that $p = p_1 + p_2 \neq 1$, and $f : E \to E'$ is a map from a norm space E into a Banach space E' such that the inequality

$$\|f(x+y) - f(x) - f(y)\| \leq \Theta\|x\|^{p_1}\|y\|^{p_2} \tag{1.5}$$

for all $x, y \in E$, then there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(x) - T(x)\| \leq \frac{\Theta}{2 - 2^p}\|x\|^p \tag{1.6}$$

for all $x \in E$. If in addition for every $x \in E$, $f(tx)$ is continuous in real t for each fixed x, then T is linear.

Suppose $(G, +)$ is an abelian group, E is a Banach space, and that the so-called admissible control function $\varphi : G \times G \to \mathbb{R}$ satisfies

$$\widetilde{\varphi}(x, y) := 2^{-1}\sum_{n=0}^{\infty} 2^{-n}\varphi(2^n x, 2^n y) < \infty \tag{1.7}$$

for all $x, y \in G$. If $f : G \to E$ is a mapping with

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y) \tag{1.8}$$

for all $x, y \in G$, then there exists a unique mapping $T : G \to E$ such that $T(x+y) = T(x) + T(y)$ and $\|f(x) - T(x)\| \leq \widetilde{\varphi}(x, x)$, for all $x, y \in G$ (see [6]).

Generalized derivations first appeared in the context of operator algebras [7]. Later, these were introduced in the framework of pure algebra [8, 9].

Definition 1.1. Let A be an algebra and let X be an A-bimodule. A linear mapping $d : A \to X$ is called

(i) derivation if $d(ab) = d(a)b + ad(b)$, for all $a, b \in A$;

(ii) generalized derivation if there exists a derivation (in the usual sense) $\delta : A \to X$ such that $d(ab) = ad(b) + \delta(a)b$, for all $a, b \in A$.

Every right multiplier (i.e., a linear map h on A satisfying $h(ab) = ah(b)$, for all $a, b \in A$) is a generalized derivation.

Definition 1.2. Let $n \geq 2$, $k \geq 3$ be positive integers. Let A be an algebra and let X be an A-bimodule. A $\mathbb{C}$-linear mapping $d : A \to X$ is called

(i) n-derivation if

$$d(a_1 a_2 \cdots a_n) = d(a_1)a_2 \cdots a_n + a_1 d(a_2)a_3 \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n) \tag{1.9}$$

for all $a_1, a_2, \ldots, a_n \in A$;

(ii) generalized (n,k)-derivation if there exists a $(k-1)$-derivation $\delta : A \to X$ such that

$$\begin{aligned} d(a_1 a_2 \cdots a_n) = {} & \delta(a_1)a_2 \cdots a_n + a_1 \delta(a_2)a_3 \cdots a_n + \cdots + a_1 a_2 \cdots a_{k-2}\delta(a_{k-1})a_k \cdots a_n \\ & + a_1 a_2 \cdots a_{k-1} d(a_k)a_{k+1} \cdots a_n + a_1 a_2 \cdots a_k d(a_{k+1})a_{k+2} \cdots a_n \\ & + a_1 a_2 \cdots a_{k+1} d(a_{k+2})a_{k+3} \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n) \end{aligned} \tag{1.10}$$

for all $a_1, a_2, \ldots, a_n \in A$.

By Definition 1.2, we see that a generalized $(2,3)$-derivation is a generalized derivation.

For instance, let $\mathcal{A}$ be a Banach algebra. Then we take

$$\mathfrak{T} = \begin{bmatrix} 0 & \mathcal{A} & \mathcal{A} & \mathcal{A} \\ 0 & 0 & \mathcal{A} & \mathcal{A} \\ 0 & 0 & 0 & \mathcal{A} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1.11}$$

$\mathfrak{T}$ is an algebra equipped with the usual matrix-like operations. It is easy to check that every linear map from $\mathcal{A}$ into $\mathcal{A}$ is a $(5,3)$-derivation, but there are linear maps on $\mathfrak{T}$ which are not generalized derivations.

The so-called approximate derivations were investigated by Jun and Park [10]. Recently, the stability of derivations have been investigated by some authors; see [10–13] and references therein. Moslehian [14] investigated the generalized Hyers-Ulam stability of generalized derivations from a unital normed algebra A to a unit linked Banach A-bimodule (see also [15]).

In this paper, we investigate the generalized Hyers-Ulam stability of the generalized (n, k)-derivations.

2. Main Result

In this section, we investigate the generalized Hyers-Ulam stability of the generalized (n, k)-derivations from a unital Banach algebra A into a unit linked Banach A-bimodule. Throughout this section, assume that A is a unital Banach algebra, X is unit linked Banach A-bimodule, and suppose that $3 \leq n$, and $3 \leq k \leq n$.

We need the following lemma in the main results of the present paper.

Lemma 2.1 (see [16]). *Let U, V be linear spaces and let $f : U \to V$ be an additive mapping such that $f(\lambda x) = \lambda f(x)$, for all $x \in U$ and all $\lambda \in \mathbb{T}^1 := \{\lambda \in \mathbb{C}; |\lambda| = 1\}$. Then the mapping f is $\mathbb{C}$-linear.*

Now we prove the generalized Hyers-Ulam stability of generalized (n, k)-derivations.

Theorem 2.2. *Suppose $f : A \to X$ is a mapping with $f(0) = 0$ for which there exists a map $g : A \to X$ with $g(0) = 0$ and a function $\varphi : A^{n+2} \to \mathbb{R}^+$ such that*

$$\begin{aligned}
&\max\{\|f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\
&\quad - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\
&\quad - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\|, \\
&\quad \|g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\
&\quad - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\|\} \\
&\quad \leq \varphi(a, b, a_1, a_2, \ldots, a_n),
\end{aligned} \tag{2.1}$$

$$\widetilde{\varphi}(a, b, a_1, a_2, \ldots, a_n) := 2^{-1} \sum_{i=0}^{\infty} 2^{-i} \varphi\left(2^i a, 2^i b, 2^i a_1, \ldots, 2^i a_n\right) < \infty \tag{2.2}$$

for all $a, b, a_1, a_2, \ldots, a_n \in A$ and all $\lambda \in \mathbb{T}^1$. Then there exists a unique generalized (n, k)-derivation $d : A \to X$ such that

$$\|f(a) - d(a)\| \leq \widetilde{\varphi}(a, a, 0, 0, 0, \ldots, 0) \tag{2.3}$$

for all $a \in A$.

Proof. By (2.1) we have

$$\begin{aligned}
&\|f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\
&\quad - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\
&\quad - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\| \\
&\quad \leq \varphi(a, b, a_1, a_2, \ldots, a_n),
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
&\| g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\
&\quad - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n \| \\
&\le \varphi(a, b, a_1, a_2, \ldots, a_n)
\end{aligned} \tag{2.5}$$

for all $a, b, a_1, a_2, \ldots, a_n \in A$ and all $\lambda \in \mathbb{T}^1$. Setting $a_1, a_2, \ldots, a_n = 0$ and $\lambda = 1$ in (2.4), we have

$$\| f(a+b) - f(a) - f(b) \| \le \varphi(a, b, 0, 0, \ldots, 0) \tag{2.6}$$

for all $a, b \in A$. One can use induction on n to show that

$$\| 2^{-m} f(2^m a) - f(a) \| \le 2^{-1} \sum_{i=0}^{m-1} 2^{-i} \varphi\left(2^i a, 2^i a, 0, 0, \ldots, 0\right) \tag{2.7}$$

for all $n \in \mathbb{N}$ and all $a \in A$, and that

$$\left\| 2^{-m} f(2^m a) - 2^{-l} f\left(2^l a\right) \right\| \le 2^{-1} \sum_{i=l}^{m-1} 2^{-i} \varphi\left(2^i a, 2^i a, 0, 0 \ldots, 0\right) \tag{2.8}$$

for all $m > l$ and all $a \in A$. It follows from the convergence (2.2) that the sequence $2^{-m} f(2^m a)$ is Cauchy. Due to the completeness of X, this sequence is convergent. Set

$$d(a) := \lim_{m \to \infty} 2^{-m} f(2^m a). \tag{2.9}$$

Putting $a_1, a_2, \ldots, a_n = 0$ and replacing a, b by $2^m a, 2^m b$, respectively, in (2.4), we get

$$\| 2^{-m} f(2^m (\lambda a + \lambda b)) - 2^{-m} \lambda f(2^m a) - 2^{-m} \lambda f(2^m b) \| \le 2^{-m} \varphi(2^m a, 2^m b, 0, 0, \ldots, 0) \tag{2.10}$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. Taking the limit as $m \to \infty$ we obtain

$$d(\lambda a + \lambda b) = \lambda d(a) + \lambda d(b) \tag{2.11}$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. So by Lemma 2.1, the mapping d is $\mathbb{C}$-linear.

Using (2.5), (2.2), and the above technique, we get

$$\begin{aligned}
&\delta(a) := \lim_{m \to \infty} 2^{-m} g(2^m a), \\
&\delta(\lambda a + \lambda b) = \lambda \delta(a) + \delta(b)
\end{aligned} \tag{2.12}$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. Hence by Lemma 2.1, δ is $\mathbb{C}$-linear. Moreover, it follows from (2.7) and (2.9) that $\|f(a) - d(a)\| \leq \tilde{\varphi}(a, a, 0, 0, \ldots, 0)$, for all $a \in A$. It is known that the additive mapping d satisfying (2.3) is unique [17]. Putting $\lambda = 1$, $a = b = 0$, and replacing $a_1, a_2, \ldots, a_n$ by $2^m a_1, 2^m a_2, \ldots, 2^m a_n$, respectively, in (2.4), we get

$$\begin{aligned} &\Big\| f(2^{nm} a_1 a_2 \cdots a_n) - 2^{(n-1)m} a_1 \cdots a_{k-1} f(2^m a_k) a_{k+1} \cdots a_n - 2^{(n-1)m} a_1 \cdots a_k f(2^m a_{k+1}) a_{k+2} \cdots a_n \\ &\quad - \cdots - 2^{(n-1)m} a_1 \cdots a_{n-1} f(2^m a_n) - 2^{(n-1)m} g(2^m a_1) a_2 \cdots a_n \\ &\quad - 2^{(n-1)m} a_1 g(2^m a_2) a_3 \cdots a_n - \cdots - 2^{(n-1)m} a_1 a_2 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \Big\| \\ &\qquad \leq \varphi(0, 0, 2^m a_1, 2^m a_2, \ldots, 2^m a_n), \end{aligned} \tag{2.13}$$

whence

$$\begin{aligned} &\| 2^{-nm} f(2^{nm} a_1 a_2 \cdots a_n) - 2^{-m} a_1 \cdots a_{k-1} f(2^m a_k) a_{k+1} \cdots a_n \\ &\qquad - 2^{-m} a_1 \cdots a_k f(2^m a_{k+1}) a_{k+2} \cdots a_n - \cdots - 2^{-m} a_1 \cdots a_{n-1} f(2^m a_n) - 2^{-m} g(2^m a_1) a_2 \cdots a_n \\ &\qquad - 2^{-m} a_1 g(2^m a_2) a_3 \cdots a_n - \cdots - 2^{-m} a_1 a_2 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \| \\ &\quad \leq 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \ldots, 2^m a_n) \end{aligned} \tag{2.14}$$

for all $a_1, a_2, \ldots, a_n \in A$. By (2.9), $\lim_{m \to \infty} 2^{-nm} f(2^{nm} a) = d(a)$ and by the convergence of series (2.2), $\lim_{m \to \infty} 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \ldots, 2^m a_n) = 0$. Let m tend to ∞ in (2.14). Then

$$\begin{aligned} d(a_1 a_2 \cdots a_n) = {} & a_1 \cdots a_{k-1} d(a_k) a_{k+1} \cdots a_n + a_1 \cdots a_k d(a_{k+1}) a_{k+2} \cdots a_n \\ & + \cdots + a_1 \cdots a_{n-1} d(a_n) + \delta(a_1) a_2 \cdots a_n + a_1 \delta(a_2) a_3 \cdots a_n \\ & + \cdots + a_1 a_2 \cdots a_{k-2} \delta(a_{k-1}) a_k \cdots a_n \end{aligned} \tag{2.15}$$

for all $a_1, a_2, \ldots, a_n \in A$.

Next we claim that δ is a $(k-1)$-derivation. Putting $\lambda = 1$, $a = b = 0$, and replacing $a_1, a_2, \ldots, a_n$ by $2^m a_1, 2^m a_2, \ldots, 2^m a_n$, respectively, in (2.5), we get

$$\begin{aligned} &\Big\| g(2^{nm} a_1 a_2 \cdots a_n) - 2^{(n-1)m} g(2^m a_1) a_2 \cdots a_n - 2^{(n-1)m} a_1 g(2^m a_2) a_3 \cdots a_n \\ &\qquad - 2^{(n-1)m} a_1 a_2 g(2^m a_3) a_4 \cdots a_n - \cdots - 2^{(n-1)m} a_1 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \Big\| \\ &\quad \leq \varphi(0, 0, 2^m a_1, 2^m a_2, \ldots, 2^m a_n), \end{aligned} \tag{2.16}$$

whence

$$\begin{aligned} &\| 2^{-nm} g(2^{nm} a_1 a_2 \cdots a_n) - 2^{-m} g(2^m a_1) a_2 \cdots a_n - 2^{-m} a_1 g(2^m a_2) a_3 \cdots a_n \\ &\qquad - 2^{-m} a_1 a_2 g(2^m a_3) a_4 \cdots a_n - \cdots - 2^{-m} a_1 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \| \\ &\quad \leq 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \ldots, 2^m a_n) \end{aligned} \tag{2.17}$$

for all $a_1, a_2, \ldots, a_n \in A$. Let m tends to ∞ in (2.17). Then

$$\begin{aligned}\delta(a_1a_2\cdots a_{k-1}a_ka_{k+1}\cdots a_n) = {}& \delta(a_1)a_2\cdots a_n + a_1\delta(a_2)a_3\cdots a_n + a_1a_2\delta(a_3)a_4\cdots a_n \\ &+\cdots + a_1a_2\cdots a_{k-2}\delta(a_{k-1})a_k\cdots a_n\end{aligned} \tag{2.18}$$

for all $a_1, a_2, \ldots, a_n \in A$.

Setting $a_k = a_{k+1} = \cdots = a_n = 1$ in (2.18). Hence the mapping δ is $(k-1)$-derivation. □

Corollary 2.3. *Suppose $f : A \to X$ is a mapping with $f(0) = 0$ for which there exists constant $\theta \geq 0$, $p < 1$ and a map $g : A \to X$ with $g(0) = 0$ such that*

$$\begin{aligned}\max\{&\|f(\lambda a + \lambda b + a_1a_2\cdots a_n) - \lambda f(a) - \lambda f(b) - a_1\cdots a_{k-1}f(a_k)a_{k+1}\cdots a_n \\ &- a_1\cdots a_kf(a_{k+1})a_{k+2}\cdots a_n - \cdots - a_1\cdots a_{n-1}f(a_n) \\ &-g(a_1)a_2\cdots a_n - a_1g(a_2)a_3\cdots a_n - \cdots - a_1a_2\cdots a_{k-2}g(a_k-1)a_k\cdots a_n\|, \\ &\|g(\lambda a + \lambda b + a_1a_2\cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1)a_2\cdots a_n \\ &-a_1g(a_2)a_3\cdots a_n - \cdots - a_1\cdots a_{k-2}g(a_{k-1})a_k\cdots a_n\|\} \\ &\leq \theta\left(\|a\|^p + \|b\|^p + \sum_{i=1}^{n}\|a_i\|^p\right)\end{aligned} \tag{2.19}$$

for all $a_1, a_2, \ldots, a_n \in A$ and all $\lambda \in \mathbb{T}$. Then there exists a unique generalized (n,k)-derivation $d : A \to X$ such that

$$\|f(a) - d(a)\| \leq \frac{\theta\|a\|^p}{1-2^{p-1}} \tag{2.20}$$

for all $a \in A$.

Proof. Put $\varphi(a, b, a_1, a_2, \ldots, a_n) = \theta(\|a\|^p + \|b\|^p + \sum_{i=1}^{n}\|a_i\|^p)$ in Theorem 2.2. □

Corollary 2.4. *Suppose $f : A \to X$ is a mapping with $f(0) = 0$ for which there exists constant $\theta \geq 0$ and a map $g : A \to X$ with $g(0) = 0$ such that*

$$\begin{aligned}\max\{&\|f(\lambda a + \lambda b + a_1a_2\cdots a_n) - \lambda f(a) - \lambda f(b) - a_1\cdots a_{k-1}f(a_k)a_{k+1}\cdots a_n \\ &- a_1\cdots a_kf(a_{k+1})a_{k+2}\cdots a_n - \cdots - a_1\cdots a_{n-1}f(a_n) \\ &-g(a_1)a_2\cdots a_n - a_1g(a_2)a_3\cdots a_n - \cdots - a_1a_2\cdots a_{k-2}g(a_{k-1})a_k\cdots a_n\|, \\ &\|g(\lambda a + \lambda b + a_1a_2\cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1)a_2\cdots a_n \\ &-a_1g(a_2)a_3\cdots a_n - \cdots - a_1\cdots a_{k-2}g(a_{k-1})a_k\cdots a_n\|\} \\ &\leq \theta\end{aligned} \tag{2.21}$$

for all $a_1, a_2, \ldots, a_n \in A$. *Then there exists a unique generalized* (n,k)*-derivation* $d : A \to X$ *such that*

$$\left\| f(a) - d(a) \right\| \leq \theta \tag{2.22}$$

for all $a \in A$.

Proof. Letting $p = 0$ in Corollary 2.3, we obtain the above result of Corollary 2.4. □

References

[1] S. M. Ulam, *Problems in Modern Mathematics*, Science Editions John Wiley & Sons, New York, NY, USA, 1964, (Chapter VI, Some Questions in Analysis: 1, Stability).

[2] D. H. Hyers, "On the stability of the linear functional equation," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 27, pp. 222–224, 1941.

[3] Th. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.

[4] Z. Gajda, "On stability of additive mappings," *International Journal of Mathematics and Mathematical Sciences*, vol. 14, no. 3, pp. 431–434, 1991.

[5] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Journal of Functional Analysis*, vol. 46, no. 1, pp. 126–130, 1982.

[6] P. Găvruţa, "A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings," *Journal of Mathematical Analysis and Applications*, vol. 184, no. 3, pp. 431–436, 1994.

[7] M. Mathieu, Ed., *Elementary Operators & Applications*, World Scientific, River Edge, NJ, USA, 1992, Proceedings of the International Workshop.

[8] F. Wei and Z. Xiao, "Generalized Jordan derivations on semiprime rings," *Demonstratio Mathematica*, vol. 40, no. 4, pp. 789–798, 2007.

[9] B. Hvala, "Generalized derivations in rings," *Communications in Algebra*, vol. 26, no. 4, pp. 1147–1166, 1998.

[10] K.-W. Jun and D.-W. Park, "Almost derivations on the Banach algebra $C^n[0,1]$," *Bulletin of the Korean Mathematical Society*, vol. 33, no. 3, pp. 359–366, 1996.

[11] M. Amyari, C. Baak, and M. S. Moslehian, "Nearly ternary derivations," *Taiwanese Journal of Mathematics*, vol. 11, no. 5, pp. 1417–1424, 2007.

[12] R. Badora, "On approximate derivations," *Mathematical Inequalities & Applications*, vol. 9, no. 1, pp. 167–173, 2006.

[13] C.-G. Park, "Linear derivations on Banach algebras," *Nonlinear Functional Analysis and Applications*, vol. 9, no. 3, pp. 359–368, 2004.

[14] M. S. Moslehian, "Hyers-Ulam-Rassias stability of generalized derivations," *International Journal of Mathematics and Mathematical Sciences*, vol. 2006, Article ID 93942, 8 pages, 2006.

[15] M. E. Gordji and N. Ghobadipour, "Nearly generalized Jordan derivations," to appear in *Mathematica Slovaca*.

[16] C.-G. Park, "Homomorphisms between Poisson JC^*-algebras," *Bulletin of the Brazilian Mathematical Society*, vol. 36, no. 1, pp. 79–97, 2005.

[17] C. Baak and M. S. Moslehian, "On the stability of J^*-homomorphisms," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 63, no. 1, pp. 42–48, 2005.