XVII. Forcing Axioms

§0. Introduction

This chapter reports various researches done at different times in the later eighties. In Sect. 1, 2 we represent [Sh:263] which deals with the relationship of various forcing axioms, mainly SPFA = MM, SPFA $\nvdash$ PFA$^+$ (=Ax_1[proper]) but SPFA implies some weaker such axioms ($\mathrm{Ax}_1[\aleph_1$-complete], see 2.14, and more in 2.15, 2.16). See references in each section.

In sections 3, 4 we deal with the canonical functions (from ω_1 to ω_1) modulo normal filters on ω_1. We show in §3 that even PFA$^+$ does not imply Chang's conjecture [even is consistent with the existence of $g \in {}^{\omega_1}\omega_1$ such that for no $\alpha < \aleph_2$ is g smaller (modulo $\mathcal{D}_{\omega_1}$) than the α-th function]. Then we present a proof that Ax[α-proper] $\nvdash$ Ax [β-proper] where $\alpha < \beta < \omega_1$, β is additively indecomposable (and state that any CS iteration of c.c.c. and $\aleph_1$-complete forcing notions is α-proper for every α).

In the fourth section we get models of CH + "ω_1 is a canonical function" without $0^\#$, using iteration not adding reals, and some variation (say ω_1 is the α-th function, CH + $2^{\aleph_1} = \aleph_3$ $|\alpha| = \aleph_2$ (see 4.7(3)). The proof is in line of the various iteration theorems in this book, so here we deal with using large cardinals consistent with $V = L$.

Historical comments are introduced in each section as they are not so strongly related.