

XVI. Large Ideals on \aleph_1 from Smaller Cardinals

§0. Introduction

We give here better consistency strength than in XIII for having some large ideal on ω_1 ; possibly without adding a real using e.g. a Woodin cardinal. By this we keep old promises from 84 – 85, mentioned in [Sh:253], Shelah and Woodin [ShWd:241], (part of the delay was because it was originally intended to be part of [ShWd:241] which later was splitted to three). This will be continued elsewhere - getting suitable axioms in 2.4, 2.5, 2.6+2.10. Woodin told the author that the results (in 2.4 – 2.6(+2.7)) threw some light on the structure of universes of set theory satisfying AD . In §2 we use from §1 only 1.2(1),(2), 1.3(1), 1.8 for 2.1; weakening somewhat the results in §2, we can use 2.8, 2.9 instead of 2.1 (so replace $(*)_{ab}^a[\lambda]$ by “ λ is a Woodin cardinal” in 2.4, 2.4A, 2.5, 2.6 thus using only 1.14, 1.15, 2.2 – 2.10).

The large cardinals from [ShWd:241] are defined in 1.14, 1.15.

§1. Bigness of Stationary $T \subseteq \mathcal{S}_{\leq \aleph_0}(\lambda)$

1.1 Notation. 1) λ a fixed regular cardinal $> \aleph_0$.

2) For sets a, b let $a \leq_\kappa b$ mean: $a \cap \kappa = b \cap \kappa$ and $a \subseteq b$ and let $a <_\kappa b$ means: