XV. A More General Iterable Condition Ensuring $\aleph_1$ Is Not Collapsed

§0. Introduction

Chapter XI was restricted to forcing notions not adding reals in a specific way so that under CH, Nm is always permissible. This was used to show that various combinatorical principles of $\aleph_2$ were equiconsistent with the existence of (small) large cardinals. We constructed our models starting from CH without adding reals, so that CH also holds in the final model. But what if we want CH to fail in the final model? Can we phrase a condition preserved by iterations, implying $\aleph_1$ does not collapse and include semiproper forcing and Nm? This, promised in the first version of this book, is carried out here. We start with notions similar to the one in Chapter X, and then move in the direction of semiproperness. Further theorems (which shed light on preservation of not adding reals) will appear elsewhere (see [Sh:311]). The preservation theorems from this chapter are sufficient to prove analogue of some theorems from Chapter XI with the negation of CH. For example adding Cohen reals to the construction of XI 1.4 we can show: If "ZFC+ $\exists$ weakly compact cardinal" is consistent, then so is "ZFC+ $2^{\aleph_0} = \aleph_2$ + for every stationary $S \subseteq S^2_0$ there is a closed copy of ω_1 included in it". Generally the preservation proofs generalize those of Chapter XI, except in the case of "iterating up to a strongly inaccessible and doing one more step (in this case 3.6). We generalize Gitik and Shelah [GiSh:191] which improve the relevant theorem in XI (i.e. [Sh:b, XI]).