# XIII. Large Ideals on $\omega_1$

## §0. Introduction

Here we shall start with $\kappa$ e.g. supercompact, use semiproper iteration to get results like ($S \subseteq \omega_1$ stationary costationary):

(a) ZFC + GCH + $\mathcal{P}(\omega_1)/(\mathcal{D}_{\omega_1} + S)$ is *layered* + suitable forcing axiom and note that by [FMSh:252] this implies the existence of a uniform ultrafilter on $\omega_1$ such that $\aleph_0^{\omega_1}/D = \aleph_1$ (which is stronger than "$D$ is not regular").

(b) ZFC+GCH+$\mathcal{P}(\omega_1)/(\mathcal{D}_{\omega_1} + S)$ is *Levy* + suitable forcing axiom.

(c) ZFC+GCH+$\mathcal{P}(\omega_1)/(\mathcal{D}_{\omega_1} + S)$ is *Ulam* + suitable forcing axiom.

where (a) Ulam means

$$(\mathcal{D}_{\omega_1} + S)^+ = \{A \subseteq \omega_1 : A \cap S \neq \emptyset \mod \mathcal{D}_{\omega_1}\}$$

is the union of $\aleph_1$, $\aleph_1$-complete filters, hence on $\mathbb{R}$ there are $\aleph_1$ measures such that each $A \subseteq \mathbb{R}$ is measurable for at least one measure

(b) Levy means that, as a Boolean algebra, it is isomorphic to the completion of a Boolean algebra of the Levy collapse Levy$(\aleph_0, < \aleph_2)$

(c) layered means that the Boolean algebra is $\bigcup_{\alpha<\aleph_2} B_\alpha$, where $B_\alpha$ are increasing, continuous, $|B_\alpha| \leq \aleph_1$, and $\mathrm{cf}(\alpha) = \aleph_1 \Rightarrow B_\alpha \lessdot \mathcal{P}(\omega_1)/(\mathcal{D}_{\omega_1} + S)$. We also deal with reflectiveness (see 4.3).

This chapter is a rerepresentation of [Sh:253], we shall give some history later, and now just remark that this work was done (and reclaimed) *after*