

XI. Changing Cofinalities; Equi-Consistency Results

§0. Introduction

We formulate a condition which is (strongly) preserved by revised countable support iteration, implies \aleph_1 is not collapsed, no real is added and is satisfied e.g. by Namba forcing, and any \aleph_1 -complete forcing. So we can iterate forcing notions collapsing \aleph_2 but preserving \aleph_1 up to some large cardinal.

Our aim is to improve the results of chapter of X to equi-consistency results. If you want to add reals, look at Chapter XV. To prove the preservation we use partition theorems and Δ -system theorems on tagged trees (3.5, 3.5A, 3.7 (and 4.3A)). Some of them are from Rubin and Shelah [RuSh:117], see detailed history there on pages 47, 48 and more on mathematics see [RuSh:117], [Sh:136] 2.4, 2.5 (pages 111 – 113).

§1. The Theorems

1.1 Discussion. In this chapter we list the demands that we would like our condition to satisfy, and show how, having a condition satisfying these demands we can prove our theorems. Then, in the following sections we will formulate the condition and prove it satisfies all our demands. Lastly we shall prove some more complicated theorems applying the condition.