IX. Souslin Hypothesis Does Not Imply "Every Aronszajn Tree Is Special"

§0. Introduction

We prove that the Souslin Hypothesis does not imply "every Aronszajn tree is special"; solving an old problem of Baumgartner, Malitz and Reinhardt. For this end we introduce variants of the notion "special Aronszajn tree" and discuss them (this is §3, see references there). We also introduce a limit of forcings bigger than the inverse limit, and prove it preserves properness and related notions not less than inverse limit, and the proof is easier in some respects, and was done already in 78; see §1, §2. We can get away without using it for the present theorems, but we want to represent it somewhere. The Aronszajn trees are addressed in §4; we choose a costationary $S \subseteq \omega_1$ and make all $\aleph_1$-trees S-st-special, while on "$\omega_1 \setminus S$ the tree remains Souslin". If $S = \emptyset$ this means that every $\aleph_1$-tree is special when restricted to some unbounded set of levels, in fact while there is no antichains whose set of levels is stationary. See more in 4.9.

§1. Free Limits

1.1 Discussion and Definitions. For A a set of propositional variables, λ a regular cardinal, let: $L_\lambda(A)$ be the set of propositional sentences generated from A, by negation and conjunction and disjunctions on sets of power $< \lambda$.