THE BOUNDARY ASSOCIATED WITH A PROBABILITY MEASURE ON A GROUP

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Let G be a locally compact group. Each probability measure, μ, on G determines a random walk on G with transition probabilities given by μ. Associated with the random walk are several different notions of 'boundary'. A discussion of the boundary is given in section 0 of [12] and some of these different notions are defined there. The Martin boundary, which is described in this volume in [13], is one of these. The boundary which is most appropriate for this paper however is the Poisson boundary which is usually much smaller than the Martin boundary (see section 0 of [12]).

The definitions of random walks and their various boundaries are not given here because we wish to describe another, more algebraic, way to associate a boundary to each probability measure μ. This boundary is Borel isomorphic to the Poisson boundary but its algebraic description provides an alternate approach to the study of boundaries. It also turns out that the boundary may be used to prove some results about $L^1(G)$. These points will be illustrated below. Details will appear in [15].

For each probability measure μ on a locally compact group G, let $J_\mu = [L^1(G) * (\delta_e - \mu)]^-$, where δ_e denotes the point mass at e, the unit element of G. Then J_μ is a closed, left ideal in $L^1(G)$ with a right, bounded approximate identity. The quotient space, $L^1(G)/J_\mu$, is therefore a left $L^1(G)$-module. It may be shown that, when it is equipped with a certain partial ordering, $L^1(G)/J_\mu$ is an abstract L^1-space and so there is a measure space (Ω, σ) such that $L^1(G)/J_\mu$ is isometric to $L^1(\Omega, \sigma)$. A G-action may be defined on Ω for which σ is quasi-invariant and so that $L^1(G)/J_\mu$ and $L^1(\Omega, \sigma)$ are isomorphic as $L^1(G)$-modules. There is also a μ-stationary probability measure, ν, which may be defined on Ω in a natural way, where