Remarks on Non-Commutative Banach Function Spaces

P.G. Dodds and B. de Pagter *

The purpose of this note is to outline an approach to the duality theory of noncommutative Banach function spaces which extends earlier work of Yeadon [Y1],[Y2]. The details will appear elsewhere.

Let \mathcal{M} be a semifinite von Neumann algebra with a semifinite normal trace τ and let $\tilde{\mathcal{M}}$ be the *-algebra of τ -measurable operators (in the sense of Nelson [N]) affiliated with \mathcal{M} . For each $x \in \mathcal{M}$ and $0 < t \in \mathbb{R}$, the generalized singular value $\mu_t(x)$ is defined to be

$$\mu_t(x) = \inf\{\lambda \ge 0 : \tau(1 - e_\lambda) \le t\}$$

where $\{e_{\lambda}\}$ denotes the spectral resolution of |x|. Our approach is based on the following result.

Proposition 1. If $x, y \in \tilde{\mathcal{M}}$, then

$$\sup\left\{\int_{E}|\mu_{t}(x)-\mu_{t}(y)|dt:|E|\leq u\right\}\leq\int_{0}^{u}\mu_{t}(x-y)dt$$

for each $u \geq 0$.

The preceding result is a common generalization of the well known inequality of Markus ([M], Theorem 5.4) for compact operators and that of Lorentz and Shimogaki [LS] for the case that \mathcal{M} is abelian. A similar inequality has been established by Hiai and Nakamura [HN] via the real interpolation method. Our present approach however is direct and is not based on interpolation methods.

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