Remarks on Non-Commutative Banach Function Spaces

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The purpose of this note is to outline an approach to the duality theory of non-commutative Banach function spaces which extends earlier work of Yeadon [Y1],[Y2]. The details will appear elsewhere.

Let $\mathcal{M}$ be a semifinite von Neumann algebra with a semifinite normal trace τ and let $\tilde{\mathcal{M}}$ be the $*$-algebra of τ-measurable operators (in the sense of Nelson [N]) affiliated with $\mathcal{M}$. For each $x \in \mathcal{M}$ and $0 < t \in \mathbb{R}$, the generalized singular value $\mu_t(x)$ is defined to be

$$\mu_t(x) = \inf\{\lambda \geq 0 : \tau(1 - e_\lambda) \leq t\}$$

where $\{e_\lambda\}$ denotes the spectral resolution of $|x|$. Our approach is based on the following result.

Proposition 1. *If $x, y \in \tilde{\mathcal{M}}$, then*

$$\sup\left\{ \int_E |\mu_t(x) - \mu_t(y)| dt : |E| \leq u \right\} \leq \int_0^u \mu_t(x - y) dt$$

for each $u \geq 0$.

The preceding result is a common generalization of the well known inequality of Markus ([M], Theorem 5.4) for compact operators and that of Lorentz and Shimogaki [LS] for the case that $\mathcal{M}$ is abelian. A similar inequality has been established by Hiai and Nakamura [HN] via the real interpolation method. Our present approach however is direct and is not based on interpolation methods.

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