ON THE PROBABILISTIC THEORY OF COMPLEX STRUCTURES

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1. Introduction

The advent of exceedingly complex electrical and mechanical systems, such as high-speed electronic computers, guidance systems for missiles, or the mechanical structure of a contemporary airplane, has made it a matter of great importance to follow up the design of such a system by some evaluation of the probability that it will perform its task without failure. If the probabilities of successful performance are known for each component of a structure, then it is usually theoretically possible to compute the probability that the entire structure will perform by carefully tracing the design step by step. This procedure may become prohibitive when the structure becomes very complex, that is, consists of a very large number of components.

It is the purpose of this report to present some results obtained by J. D. Esary, S. C. Saunders, and the writer, which deal with certain properties of complex structures and their reliabilities, and either do not depend on the number of components, or else display an asymptotic behavior as the number of components increases. In section 2, a number of properties of structures will be discussed which may be described as combinatorial and which, while interesting in themselves, are preliminary to a probabilistic discussion of structure reliability. In section 3 the probability of successful performance will be studied for the kinds of structures analyzed in section 2. A number of our results are generalizations ideas of originated by von Neumann [1] and systematically developed by Moore and Shannon [2]. The present report is meant to give a survey of the main findings and does not contain the mathematical derivations of the theory. A detailed mathematical presentation is contained in a manuscript submitted for publication [3].

2. Structures and their combinatorial properties

2.1. *Dichotomic structures.* We will limit our presentation to devices (components as well as systems built of components) which can be in only one of two states: they either perform or fail. The state will be described by the value of an indicator variable which will be given the value 1 when the device performs and

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