BOUNDARY REGULARITY FOR SOLUTIONS OF THE EQUATION OF PRESCRIBED GAUSS CURVATURE

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We describe a recent boundary regularity result for the equation of prescribed Gauss curvature

$$\det D^2u = K(x)(1+|Du|^2)^{(n+2)/2} \tag{1}$$

in the case that the gradient of the solution is infinite on some relatively open portion of the boundary of the domain. To see how this situation arises, we recall that to solve the Dirichlet problem for (1) on a smooth, uniformly convex domain $\Omega \subset \mathbb{R}^n$ we need two conditions on the function K. First, we need

$$\int_\Omega K < \omega_n , \tag{2}$$

where ω_n is the measure of the unit ball in $\mathbb{R}^n$, to obtain a bound for the maximum modulus of the solution in terms of its boundary values, and second, we need

$$K(x) \leq \mu \operatorname{dist}(x,\partial\Omega) \tag{3}$$

for some positive constant μ to obtain a boundary gradient estimate. We then have the following theorem (see [4]).

THEOREM 1 *Let* Ω *be a* $C^{1,1}$ *uniformly convex domain in* $\mathbb{R}^n$ *and let* $K \in C^{1,1}(\Omega)$ *be a positive function satisfying* (2) *and* (3). *Then the Dirichlet problem*

$$\begin{aligned} \det D^2u &= K(x)(1+|Du|^2)^{(n+2)/2} \quad \text{in } \Omega , \\ u &= \varphi \quad \text{on } \partial\Omega \end{aligned} \tag{4}$$

has a unique convex solution $u \in C^2(\Omega) \cap C^{0,1}(\bar{\Omega})$ *for every* $\varphi \in C^{1,1}(\partial\Omega)$.