CRYSTALLINE GEOMETRIC CRYSTAL GROWTH

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In geometric crystal growth, the normal velocity of the interface is the product of the mobility of the interface in that normal direction with the force driving the crystal to grow. This force is often a constant Ω, representing the decrease in free energy per unit volume due to the crystal growing, plus the "weighted mean curvature," representing the change in surface energy per unit volume swept out by the growth. A survey of nine mathematical formulations of such motion is in preparation [TCH].

This paper is a progress report on the "crystalline" formulation, in which the surface free energy γ is so highly anisotropic that the equilibrium crystal shape

$$W = \{\mathbf{x} : \mathbf{x} \cdot \mathbf{n} \leq \gamma(\mathbf{n}) \text{ for every unit vector } \mathbf{n}\}$$

is a polyhedron, with its set of normals equal to a specified finite set $\mathcal{N}$. It was first introduced in [T1] and was discussed in [T2].

Crystalline geometric crystal growth is studied for three reasons: (1) sometimes physical problems require it, (2) it ought to be useful as a natural polyhedral approximation to more smooth motions, and (3) it has an interesting formulation and theory of its own, in which curvature is not defined at points but on line or plane segments. Results have been proved which sometimes parallel the results for motion by mean curvature and which sometimes are strikingly different [T3]. Also, a computer program has been written which computes this motion, for immersed as well as embedded and multiple-grain-junction curves in the plane (with and without fixed boundaries), and another program is under development for surfaces. These programs are illustrated in the a videotape which is part of [T4]. (A very early version of this program was demonstrated in [T2].) A related program for the motion of curves in 2-d was developed by Roberts and is described in [R].