

Moduli of stable parabolic connections, Riemann-Hilbert correspondence and geometry of Painlevé equation of type VI, part II

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*Dedicated to Professor Masaki Maruyama
on his 60th birthday*

Abstract.

In this paper, we show that the family of moduli spaces of α' -stable (t, λ) -parabolic ϕ -connections of rank 2 over \mathbf{P}^1 with 4-regular singular points and the fixed determinant bundle of degree -1 is isomorphic to the family of Okamoto–Painlevé pairs introduced by Okamoto [O1] and [ST1]. We also discuss about the generalization of our theory to the case where the rank of the connections and genus of the base curve are arbitrary. Defining isomonodromic flows on the family of moduli space of stable parabolic connections via the Riemann-Hilbert correspondences, we will show that a property of the Riemann-Hilbert correspondences implies the Painlevé property of isomonodromic flows.

§1. Introduction

In part I [IIS1], we established a complete geometric background for Painlevé equations of type VI or more generally for Garnier systems from view points of moduli spaces of rank 2 stable parabolic connections, moduli spaces of SL_2 -representations of $\pi_1(\mathbf{P}^1 \setminus D(t))$ and the Riemann-Hilbert correspondences between them.

In this formulation, Painlevé equations of type VI or Garnier systems are vector fields or systems of vector fields on each corresponding family of moduli spaces of stable parabolic connections arising from

Received December 5, 2005.

¹Partly supported by Grant-in Aid for Scientific Research (Wakate-B-15740018).

²Partly supported by Grant-in Aid for Scientific Research (B-12440043).

³Partly supported by Grant-in Aid for Scientific Research (B-16340009), (Houga-16654004), and JSPS-NWO exchange program.