BANACH ALGEBRA TECHNIQUES AND EXTENSIONS OF OPERATOR-VALUED REPRESENTATIONS

B. Chevreau and J. Galé *

1. INTRODUCTION

The existence of a functional calculus associated to a (bounded linear) operator T on a (complex) Banach space E can be very useful in the study of T, provided this functional calculus is defined on a sufficiently rich class of functions. In this note we consider several situations where standard Banach algebra techniques (mainly the use of a bounded approximate identity via Cohen's factorization theorem for modules) lead to extensions of a given functional calculus to a larger algebra. The typical case we discuss (§3) is that of a representation Φ of the standard disc algebra $\mathcal{A}(\bar{\mathbf{D}})$ into the Banach algebra $\mathcal{L}(E)$ of operators on E. (Indeed, any contraction on a Hilbert space gives rise to such a representation via von Neumann's inequality.) In this situation we can extend Φ to subalgebras H^∞_Γ of H^∞ (see below for definitions) where Γ is an open subset of the unit circle whose complement in $\mathbf{T}$ has (Lebesgue) measure 0. It turns out that such algebras have already been considered in the literature (cf. [3]). We conclude this section with an investigation of the "maximal" extension that can be obtained in this fashion.

In section 4 we discuss the same problem where the disc algebra is replaced by an arbitrary function algebra $\mathcal{A}$. Particular cases of this situation had already been studied in [1]. Here the "leading thread" is the connection between peak sets of $\mathcal{A}$ and bounded approximate identities for certain ideals of $\mathcal{A}$.

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