and

$$\dim(A/\mathfrak{p}) \leqq \dim(A).$$

The latter is geometrically interpreted as follows: If $x \in \overline{y}$, then $\dim(V(j_x)) \leqq \dim(V(j_y))$.

Dimension is a very coarse invariant, i.e. were we to consider the equivalence classes of affine varieties of a given dimension, we would obtain huge classes of highly non isomorphic varieties.

§3. DEPTH

The next numerical invariant we shall study in the notion of depth. We assume throughout this section that A is a noetherian local ring with maximal ideal $\mathfrak{m}$, and that M is a finitely generated A-module.

Definition 3.1. a) an element $x \in A$ is called M-regular if the homomorphism $\varphi: M \to M$ given by $\varphi(m) = xm$ is injective.

b) a sequence $\{x_1,\ldots,x_n\}$ of elements of A is called M-regular if x_i is $M/x_1 M + \ldots + x_{i-1} M$ regular, $1 \leqq i \leqq n$.

Remark. Clearly every $x \notin \mathfrak{m}$ being invertible is M-regular for every module M. Hence we shall confine our attention to those M-regular elements which belong to $\mathfrak{m}$. With regard to b) we state, without proof, the fact that the sequence $\{x_1,\ldots,x_n\}$ is M-regular if, and only if all sequences $\{x_{\sigma(1)},\ldots,x_{\sigma(n)}\}$ $\sigma \in S_n$ are M-regular, where S_n denotes the group of permutations on n symbols. (Grothendieck, E.G.A., Ch. 0, §15.1, I.H.E.S. no 20) The above statement is false if A is not noetherian.