arbitrary closed subsets by the formula

$$\dim(V(\mathfrak{a})) = \dim(A/\mathfrak{a})$$

where $\mathfrak{a}$ is an arbitrary ideal of A.

If M is a finitely generated A-module we define

$$\dim(M) = \dim(\mathrm{Supp}(M)) = \dim(A/\mathrm{ann}(M)).$$

Here we use the fact, mentioned in the preliminaries, that Supp(M) is the closure in Spec(A) of Ass(M), and Ass(M) consists of the prime ideals associated to ann(M).

If $N \subset M$ is another A-module we see trivially that

$$\dim(N) \leqq \dim(M)$$

$$\dim(M/_N) \leqq \dim(M)$$

In fact $\mathrm{ann}(N) \supset \mathrm{ann}(M)$, $\mathrm{ann}(M/_N) \supset \mathrm{ann}(M)$.
A non-trivial statement, proved in Bourbaki's, chapter IV, §2, is the following:

Theorem 1.2. $\dim(M) = 0$ if, and only if, M has finite length, in the composition series sense.

§2. HILBERT-SAMUEL POLYNOMIAL

Let H be a graded ring, i.e.

$$H = \bigoplus_{n \geqq 0} H_n$$

where H_n are (additive) groups and $h_n \cdot h_m \in H_{n+m}$, for $h_n \in H_n$, $h_m \in H_m$. Clearly H_n is an H_0-module. We assume:

a) H_0 is an artinian ring

b) H is generated (as an H_0-algebra) by finitely many elements of H_1.