My interest in two of the main subjects of these lectures goes back to my student days at Frankfurt (1923-25) and Gottingen (1925-33). From C. L. Siegel I learned of Thue's theorem and its improvements and generalisations; and Emmy Noether introduced me to the theory of p-adic numbers. I combined these two ideas in 1931 when I found an analogue of the Thue-Siegel theorem that involved both real and p-adic algebraic numbers. In later work I repeatedly came back to such problems, and already in 1934-36 I gave a course on Diophantine approximations at the University of Groningen dealing with problems that simultaneously involved the real and p-adic fields.

After the war, E. Lutz published her very beautiful little book on Diophantine approximations in the p-adic field. But she considered alone the case of numbers in one fixed p-adic field.

In 1955, K. F. Roth obtained his theorem on the rational approximations of a real algebraic number. It was immediately clear that his method should also work for p-adic algebraic numbers, for Roth's method could clearly be combined with that of my old papers. Some interesting work of this kind was in fact carried out by D. Ridout, a student of Roth. In the second part of these lectures I shall try to go rather further in this direction. As the proofs will show, the p-adic numbers, and more generally the g-adic numbers form an important tool in these investigations; but one form of the final result will be again free of such numbers and will state a property of rational numbers only.

The first part of these lectures has mainly the purpose of acquainting the reader with the theory of p-adic and g-adic numbers. It gives a short account of the theory of valuations, and I have found it convenient to discuss also the slightly more general theory of pseudovaluations because it leads in a very natural way to Hensel's g-adic numbers. The results in Chapters 3 and 4 serve chiefly as examples, but have perhaps also a little interest in themselves.

The whole second part, as well as two short appendices, deal with a very general g-adic form of Roth's theorem. As the proof is rather involved, all details are given, and I have also tried to explain the reasons behind the different steps of the proof.

The most original part of Roth's proof consists in a very deep theorem, here called Roth's Lemma. It states that, under certain conditions, a polynomial in a large number of variables cannot have a multiple zero of too high an order.

Since Roth's Lemma is essentially a theorem on the singularities of an algebraic manifold, perhaps the methods of algebraic geometry may finally enable one to obtain a simpler proof and a stronger result (say, with the upper bound $2^{m+1}\, t^{2^{-(m-1)}}$ in Roth's Lemma replaced by something like $t^{m^{-c}}$). It would then become possible to improve the theorem by Cugiani given in the appendix.

Another possible approach to Roth's Lemma is from the theory of