$$\{a_0, b_0, c_0, \ldots\},$$

where $a_0 \in A' - A_1$, $b_0 \in B' - B_1$, ... However this element cannot correspond to any element of ST. Indeed it cannot be mapped on an element of A_0, for example, because if it could, a_0 would have to be one of the elements of A_1.

4. The well-ordering theorem

After all this I shall now prove, by use of the choice principle, that every set can be well-ordered. First I shall give another version of the notion "well-ordered", different from the usual one.

We may say that a set M is well-ordered, if there is a function R, having M as domain of the argument values and UM as domain of the function values, such that if $N \supset 0$ is arbitrary and $\in$ UM, there is a unique $n \in N$ such that $N \subseteq R(n)$. I have to show that this definition is equivalent to the ordinary one. If M is well-ordered in the ordinary sense, then every non-void subset N has a unique first element. Then it is clear that if $R(n)$, $n \in M$, means the set of all $x \in M$ such that $n \leqq x$, the other definition is fulfilled by this R. Let us, on the other hand, assume that we have a function R of the said kind. Letting N be $\{a\}$, one sees that always $a \in R(a)$. Let N be $\{a,b\}$, $a \neq b$. Then either a or b is such that $N \subseteq R(a)$ resp. $R(b)$. If $N \subseteq R(a)$, then we put $a < b$. Since then N is not $\subseteq R(b)$, we have $a \bar{\in} R(b)$. Now let $b < c$ in the same sense that is, $c \in R(b)$, $b \bar{\in} R(c)$. Then it is easy to see that $a < c$. Indeed we shall have $\{a,b,c\} \subseteq$ either $R(a)$ or $R(b)$ or $R(c)$, but $b \bar{\in} R(c)$, $a \bar{\in} R(b)$. Hence $\{a,b,c\} \subseteq R(a)$ so that $\{a,c\} \subseteq R(a)$, i.e. $a < c$. Thus the defined relation $<$ is linear ordering. Now let N be an arbitrary subset of M and n be the element of N such that $N \subseteq R(n)$. Then if $m \in N$, $m \neq n$, we have $m \in R(n)$, which means that $n < m$. Therefore the linear ordering is a well-ordering.

Theorem 10. *Let a function ϕ be given such that $\phi(A)$, for every A such that $O \subset A \subseteq M$, denotes an element of A. Then UM possesses a subset $\mathfrak{M}$ such that to every $N \subseteq M$ and $\supset O$ there is one and only one element N_0 of $\mathfrak{M}$ such that $N \subseteq N_0$ and $\phi(N_0) \in N$.*

Proof: I write generally $A' = A - \{\phi(A)\}$. I shall consider the sets $P \subseteq$ UM which, like UM, possess the following properties

1) $M \in P$

2) $A \in P \rightarrow A' \in P$ for all $A \subseteq M$

3) $T \quad P \rightarrow DT \in P$.

These sets P constitute a subset $\mathfrak{T}$ of UUM. They are called Θ-chains by Zermelo. I shall show that the intersection $D\mathfrak{T}$ of all elements of $\mathfrak{T}$ is again a Θ-chain, that is, $D\mathfrak{T} \in \mathfrak{T}$. It is seen at once that $D\mathfrak{T}$ possesses the properties 1) and 2). Now let $T \subseteq D\mathfrak{T}$. Then, if $P \in \mathfrak{T}$, we have $T \subseteq P$, and since 3) is valid for P, also $DT \in P$. Since this is true for all P, we have $DT \in D\mathfrak{T}$ as asserted. Thus I have proved that $D\mathfrak{T} \in \mathfrak{T}$.