

Foreword

This book deals with infinite-dimensional Kähler manifolds, more precisely, with three particular examples of such manifolds — loop spaces of compact Lie groups, Teichmüller spaces of complex structures on loop spaces, and Grassmannians of Hilbert spaces. There is an opinion that there could not be a comprehensive theory of Kähler manifolds in the infinite-dimensional setting. Such an opinion is based on the belief that infinite-dimensional Kähler manifolds are too rich and too different from each other so that any of them deserves its own theory. It's hard to say now whether a general theory of infinite-dimensional Kähler manifolds may or may not exist but it is certainly true that each of our three examples deserves a separate study. Any of these manifolds can be considered as a universal object in a certain category, containing all its finite-dimensional counterparts. In particular, main ingredients of Kähler geometry of these finite-dimensional spaces may be recovered from the corresponding ingredients, attached to the universal object, by restriction. Therefore, one can expect that it may be more natural and sometimes easier to study these ingredients for the universal object, rather than for its finite-dimensional counterparts. We'll give several examples of this sort in our book, and I'm sure that many more are to be found in future.

The choice of the three infinite-dimensional Kähler spaces for our study is, by no means, accidental. It is motivated by the relation of these spaces to various problems in modern mathematical physics. We do not consider these intriguing relations in our book in order to save its volume with only one exception. Since our first interest in infinite-dimensional Kähler manifolds emerged from the geometric quantization of loop spaces (related to string theory), we could not refuse ourselves in supplying the book with a second part, devoted to this subject (together with a brief survey of the geometric quantization of finite-dimensional Kähler manifolds).

My interest in the geometric quantization of infinite-dimensional phase manifolds arose from reading the papers by Bowick–Rajeev [14] and Kirillov–Yuriev [44]. I began to study the Pressley–Segal treatise on loop spaces [65], which became my handbook on this subject and infinite-dimensional Kähler manifolds, in general. The current edition may be considered as an attempt, inspired by [65], to expose in a concise form geometric ideas, lying behind the loop space theory. It should be also mentioned here a stimulating paper by Nag–Sullivan [58], which has revealed the role of the universal Teichmüller space and the Sobolev space of half-differentiable functions on the circle for the geometric quantization of loop spaces and string theory.