RE in $\mathbf{a}$ such that neither $\mathbf{b} \leq \mathbf{c}$ nor $\mathbf{c} \leq \mathbf{b}$. The method used to define $\mathbf{0}'$, when relativized, shows that there is a largest degree RE in $\mathbf{a}$. It is called the jump of $\mathbf{a}$, and is designated by $\mathbf{a}'$. The relativized Limit Lemma shows that a real is a limit of a recursive in $\mathbf{a}$ sequence of reals iff it has degree $\leq \mathbf{a}'$.

16. Evaluation of Degrees

We shall now show how to evaluate the degrees of certain explicitly given relations.

Let Φ be a class of relations. We say a relation R is Φ complete if R is in Φ and every relation in Φ is reducible to R (where reducible is defined before 13.3). It follows that R has the largest degree of any relation in Φ; so any two Φ complete relations have the same degree. (Caution: Some authors use complete in a somewhat different way.)

EXAMPLE. If F is total, $W_e^F(x)$ is RE in F complete; its degree is the jump of dg F. Hence any RE in F complete relation has degree (dg F)'.

The degree obtained by applying the jump n times to $\mathbf{0}$ is designated by $\mathbf{0}^n$.

16.1. PROPOSITION. For every n, there is a Σ_n^0 complete set of degree $\mathbf{0}^n$ and a Π_n^0 complete set of degree $\mathbf{0}^n$.

Proof. We use induction on n. If $n = 1$, let P be a recursive set; if $n > 1$, let P be a Π_{n-1}^0 complete set of degree $\mathbf{0}^{n-1}$. Then $W_e^P(x)$ has degree $\mathbf{0}^n$ by the example. By Post's Theorem, Σ_n^0 is the class of relation RE in P; so $W_e^P(x)$ is Σ_n^0 complete. Then $\neg W_e^P(x)$ is of degree $\mathbf{0}^n$ and is Π_n^0 complete. □

16.2. COROLLARY. Every Σ_n^0 complete or Π_n^0 complete relation has degree $\mathbf{0}^n$. □

If Φ is a class of RE sets, then the set of indices of sets in Φ is called the index set of Φ.

16.3. PROPOSITION (RICE). If Φ is a non-empty class of RE sets which is