ABOUT THE IRREFLEXIVITY HYPOTHESIS
FOR FREE LEFT DISTRIBUTIVE MAGMAS

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Abstract. An important combinatorial statement about free left distributive structures, the irreflexivity hypothesis, has been proved by R. Laver using a large cardinal axiom. We discuss here another approach that could, if completed, lead to a new proof independent of any set theoretical assumption.

A left distributive magma—or LD-magma—will be any set endowed with a binary law satisfying the left distributivity identity

$$x(yz) = (xy)(xz).$$

The interest for (free) LD-magmas was emphasized by the study of the iterations of an elementary embedding of a rank into itself in set theory and the conjecture that the structure obtained in this way is actually a free (monogenic) LD-magma. This conjecture has been proved in 1989 by Richard Laver ([La]); an alternative proof in given in [De4]. Both proofs make an intensive use of the relation of being a left factor in LD-magmas.

DEFINITION. Let $\mathfrak{g}$ be a LD-magma, and x, y belong to $\mathfrak{g}$; write $x <_L^{\mathfrak{g}} y$ if, and only if, there exists a (positive) integer p and a finite sequence $z_1, z_2, \ldots, z_p$ in $\mathfrak{g}$ such that y is equal to $(\cdots((xz_1)z_2)\cdots)z_p$.

The statement we shall discuss here is the

IRREFLEXIVITY HYPOTHESIS (IH). *Let $\mathfrak{f}$ be the free monogenic LD-magma; then $<_L^{\mathfrak{f}}$ is an irreflexive relation.*

This property proved to be crucial in the study of free LD-magmas. In particular, the following was proved independently in [De2], and in [La1] (for the monogenic case):

PROPOSITION. *If IH is true, then for any set Σ, the word problem for the free LD-magma generated by Σ is decidable; also every free LD-magma admits left cancellation.*