Kurt Gödel and the Consistency of $\mathbf{R}^{\#\#}$*

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Summary. This paper continues my investigations of arithmetics formulated relevantly. (See references [1], [10], [12], and [9].) It is proved again that relevant **Peano** arithmetic $\mathbf{R}^{\#}$ and relevant **true** arithmetic $\mathbf{R}^{\#\#}$ (with the ω-rule) are **demonstrably consistent** by simple finitary arguments. E.g., it requires little more than truth-tables to show that $0 = 1$ is a non-theorem. This **removes** much of the sting from Gödel's **second** theorem. Regard for relevance **bounds** the harm that even potential contradictions can do. But Gödel still collects his **dues**, since proving **negation-consistency** remains annoyingly (and ineluctably) **non-constructive**. To the extent that $\sim$ in *Formalese* is unlike **not** in *English*, as it seems to be, Gödel's theorems are **dirty tricks**.

1.

Being mainly self-educated in beginning logic, I was alarmed to read in [2] that Gödel had shown that elementary number theory is either inconsistent or incomplete. "What," thought I to myself, "could this possibly **mean**?" Could it be in **doubt** that 2+2 = 4? Might one **multiply** 27 and 37 and get 998? What is **going on** here?

More mature reflection convinces one that what is going on is a logical **dirty trick**. Speaking at his most persuasive, Gödel in [3] conned a certain sentence G into saying of itself that it was unprovable.[1] We all know where the story goes from there, at least intuitively. If G is *false*, then it is provable after all, which engenders contradiction. So G had better be true. And, as this reasoning can be carried out in any sufficiently strong, consistent and effective system **S**, the moral **is** (or is **alleged** to be) that

(I) **S** is incomplete, containing an unprovable truth (Gödel's **first** theorem), and

(II) **S** lacks the means to formalize a proof of **its own** consistency (Gödel's **second** theorem).

I shall throw no stones at (I) here. But (II) is another matter. The idea behind it is said to be (in, e. g., [3] and [4]) that we can **formalize** the proof of (I). This leaves us with the following **S**-theorem:[2]

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* This paper is in its final form and no similar paper has been or is being submitted elsewhere.

[1] G is 17 gen r, says [3].

[2] We follow [3] in using "Wid" (for "widerspruchsfrei") for "consistent".