Forcing on Bounded Arithmetic

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Forcing method on Bounded Arithmetic was first introduced by J. B. Paris and A. Wilkie in [10]. Then M. Ajtai started in [1], [2] and [3] elaborate use of the method to get excellent results on the pigeon hole principle and the module p counting principles. Ajtai's work were followed by many works by Beame et als, Krajíček and Riis in [4], [5], [8], [9], [11].

In this paper, we develop a Boolean valued version of forcing on Bounded Arithmetic using big Boolean algebra, and discuss its relation with $NP = co-NP$ problem and $P = NP$ problem.

As is well known, Gödel raised the problem closely related to $P = NP$ problem in his letter to von Neumann in 1956. We believe that Gödel would greatly contribute to it if the complexity theory would have started at the time.

We also would like to mention about Gödel's close felling to Boolean valued models. Forcing and Boolean valued model theory are equivalent. But Gödel was much more impressed by Boolean valued models than forcing in the following reason. Gödel did have a systematic reinterpretation of the logical operations with a view to a formal independence proof, but it was too messy for his taste. He realized that the Boolean valued models are a straightforward model-theoretic variant of his earlier reinterpretation.

When one of the authors started Boolean valued analysis by using Boolean algebras of projections in Hilbert space, he received a strong encouragement from Professor Gödel. We feel that our work is in the line of Gödel's vision.

1. The generic models

Let N be a countable nonstandard model of the true arithmetic $Th(\mathbb{N})$ where $\mathbb{N}$ is the standard model of arithmetic. Let n be a nonstandard element in N and $M = \{x \in N \mid \text{there exists some } n\#\cdots\#n \text{ such that } x \leq n\#\cdots\#n\}$.

* This is the final version of the paper which will not be published elsewhere.