Towards Recursive Model Theory*

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Abstract. We argue that the models that are relevant to computer science are recursive and that Recursive Model Theory deserves being studied systematically, with at least the same vigor as Finite Model Theory has been. We study the status of some fundamental theorems from the classical model theory in this context and establish failure of several of them, including (generalized) Completeness, Compactness, Beth's Definability, Craig's Interpolation, and Lyndon's Lemma.

1 Introduction

Classical Model Theory deals with *all models.* If, for whatever reason, the class of models is restricted, this may potentially change model-theoretical laws that we take for granted. Take, for instance, the central for logic notion of *truth.* There may be sentences that are uniformly true in all the models of a certain class, but refutable in models not in the considered class. Hence, restricting the class of models may expand the class of true sentences. Conversely, the class of satisfiable sentences may shrink. If this actually happens, the equivalence between Model Theory and Proof Theory implied by Gödel's Completeness Theorem discontinues to hold, although in a specific situation a remedy can possibly be found by changing the axiomatization.

Recently, the Model Theory of finite models (those with finite universes) has been intensively investigated. The main motivation for Finite Model Theory has been the fact that, in several computer science applications, notably in databases, the models are often finite, and many issues in the theory of databases can be studied in the context of Finite Model Theory. Surprisingly or not, Finite Model Theory looks very much different from its classical counterpart.

The author is generally interested in Logic in Computer Science, and while finite models often are relevant to Computer Science, without question, not all the models that show up in CS applications are finite. Even in databases which have long been the Finite Model Theory refuge, infinite models not only show up, but actually move towards the central stage. In other CS playgrounds, say, in verification, finite models have never had any noticeable fraction of the market.

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