THE USE OF THE LIKELIHOOD FUNCTION IN STATISTICAL PRACTICE

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1. Introduction

The title I have chosen deliberately echoes that of the paper by L. J. Savage, [1] because it is written with an objective which closely corresponds to Savage's —to encourage practical statisticians to explore the ways in which the study of the likelihood function generated by a set of data can help in its interpretation; At the same time I hope theoretical statisticians will be encouraged to study the theory of likelihood with a view to explaining in detail how the likelihood function can be used, and what its limitations are. It appears to be high time we did this, because for some years now it has been common for geneticists to express themselves in terms of likelihood, and the following quotation indicates that high energy physicists are following suit: "How then can an experimenter present the results of his work in an 'objective' fashion, that is, without introducing his own prior beliefs? One way, *often used by physicists* (my italics, G.B.), is to present $L(x_i^{obs}; \alpha)$ as a function of α for his particular observations $x_i^{obs}; \ldots$" [2].

Whereas in some fields it is still possible to attribute failure to use "orthodox" statistical methods to mere ignorance, such an explanation is untenable in the case of statistically sophisticated areas such as these two. It must be here that the "orthodox" methods have been tried and found wanting.

Lest my allusion to Savage be misinterpreted to mean that I accept the subjective Bayesian position, let me hasten to specify some of the ways in which we differ. Whereas Savage, if I understand him aright, would regard a specification of the likelihood function as *always* providing, at least in principle, the solution to problems of statistical inference, I conceive of likelihood methods as rigorously applicable only to those situations where the distribution of the observations x over the sample space S can be taken as known to belong to a (usually continuously) parametrized family of distributions with probability functions $f(x, \theta)$, the parameter θ ranging over a well-defined parameter space Ω. The primary problem in such a case is often how to express the order of preference among the different values of θ which may be said to be rationally induced

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