A STOCHASTIC DESCRIPTION OF PRECIPITATION

L. LE CAM
UNIVERSITY OF CALIFORNIA, BERKELEY

1. Introduction

During the early part of the war years Pierre Massé organized a group for the purpose of studying optimal procedures of development and management of the French hydroelectric and steam power system. The problems encountered by this group included the evaluation of probabilities of excessive discharges, the evaluation of probabilities of excessive droughts, as well as the development of optimal management procedures for the big and small hydroelectric reservoirs.

As the studies of the group progressed, the need for a mathematically tractable description of the random structure of stream flow became more and more imperative. To obtain such a description it was found necessary to start with a description of the random structure of rainfall. The purpose of the present paper is to give a summary account of such a description.

Tentative descriptions of rainfall behavior at one raingage station were introduced around 1944 by M. Loève and independently by E. Halphen. Related formulations have been used more recently by P. A. P. Moran [1] in the study of dams. See also Gani [2] and D. G. Kendall [3]. A description of the areal and temporal behavior of precipitation was introduced by the author around 1947 as an aid in the study of peak discharges. This description, which will be detailed below, does not actually qualify as a stochastic model for precipitation.

A true model should take into account the applicability of the laws of fluid mechanics and thermodynamics. The description given below does not make any provision for the introduction of relations between winds, temperature, origin of air masses, and so forth, and the precipitation itself. Although we have recently attempted to make use of whatever meteorological considerations were accessible to us, at the time of this writing we have not yet met with any reportable success. However there is some hope that our goal will become more attainable in the near future. Also, we hope that the mathematical technique used here will remain applicable in some realistic studies.

Section 2 gives an informal description of the behavior of rainfall that will be converted into formulas in section 5. Sections 3 and 4 introduce the mathemathical apparatus necessary for the conversion.

It is to be noted that the model exposed here is essentially a clustering process

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