CHAPTER 1

MATHEMATICS = SET THEORY?

> *"No one shall drive us out of the paradise that Cantor has created"*
>
> David Hilbert

1.1. Set theory

The basic concept upon which the discipline known as *set theory* rests is the notion of *set membership.* A set may be initially thought of simply as a collection of objects, these objects being called *elements* of that collection. *Membership* is the relation that an object bears to a set by dint of its being an element of that set. This relation is symbolised by the Greek letter $\in$ (epsilon). "$x \in A$" means that A is a collection of objects, one of which is x, i.e. x is a *member* (element) of A. When x is not an element of A, this is written $x \notin A$. If $x \in A$, we may also say that x *belongs* to A.

From these fundamental ideas we may build up a catalogue of definitions and constructions that allow us to specify particular sets, and construct new sets from given ones. There are two techniques used here.

(a) *Tabular form:* this consists in specifying a set by explicitly stating all of its elements. A list of these elements is given, enclosed in brackets. Thus

$$\{0, 1, 2, 3\}$$

denotes the collection whose members are all the whole numbers up to 3.

(b) *Set Builder form:* this is a very much more powerful device that specifies a set by stating a property that is possessed by all the elements of the set, and by no other objects. Thus the property of "being a whole number smaller than four" determines the set that was given above in tabular form. The use of properties to define sets is enshrined in the

PRINCIPLE OF COMPREHENSION. *If $\varphi(x)$ is a property or condition pertaining to objects x, then there exists a set whose elements are precisely the objects that have the property (or satisfy the condition) $\varphi(x)$.*