Chapter 16

INVERSIONS IN CIRCLES

Q: How does a geometer capture a lion in the desert?

A: Build a circular cage in the desert, enter it, and lock it. Now perform an inversion with respect to the cage. Then you are outside, and the lion is locked in the cage.

— A mathematical joke from before 1938

We will now study inversions in a circle (which is the analogue of reflection in a line) and its applications. Though inversion in a circle can be defined on spheres and hyperbolic planes, it seems to have no significant applications on these surfaces. Therefore, in this chapter we will only consider the case of the Euclidean plane.

To study Chapter 16, the only results needed in Chapters 10–15 are

PROBLEM 13.4: The **AAA similarity** and **SAS similarity** criteria for triangles on the plane.

PROBLEM 15.1b: *On a plane, if two lines through a point P intersect a circle at points A, A' (possibly coincident) and B, B' (possibly coincident), then* $|PA| \times |PA'| = |PB| \times |PB'|$.

If you are willing to assume these criteria for similar triangles, then you can work through Chapter 16 without Chapters 10–15.

EARLY HISTORY OF INVERSIONS

Apollonius of Perga (c. 250–175 B.C.) was famous in his time for work on astronomy (Navigation/Stargazing Strand), before his now well-known work on conic sections.