

## Chapter 7

# AREA AND HOLONOMY



We [my student and I] are both greatly amazed; and my share in the satisfaction is a double one, for he sees twice over who makes others see.

— Jean Henri Fabre, *The Life of the Fly*, New York: Dodd, Mead and Co., 1915, p. 300.

There are many things in this chapter that have amazed us and our students. We hope you, the reader, will also be amazed by them. We will find a formula for the area of triangles on spheres and hyperbolic planes. We will then investigate the connections between area and *parallel transport*, a notion of local parallelism that is definable on all surfaces. We will also introduce the notion of *holonomy*, which has many applications in modern differential geometry and engineering.

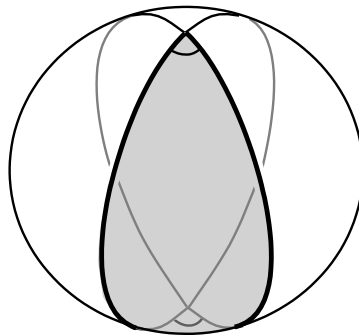


Figure 7.1 Lune or biangle

**DEFINITION:** A *lune* or *biangle* is any of the four regions determined by two (not coinciding) great circles (see Figure 7.1).

The two angles of the lune are congruent. (*Why?*)