CHAPTER 1

Definitions of Transversely Holomorphic Foliations and Complex Secondary Classes

1.1. Basic Notions

In this monograph, foliations are assumed to be regular (without singularities) unless otherwise mentioned.

Definition 1.1.1. Let M be a manifold without boundary. A decomposition of M into immersed submanifolds $\{L_\alpha\}_{\alpha\in A}$, called *leaves*, is a *foliation* of M if there is an integer q and an atlas $\{U_\lambda\}_{\lambda\in\Lambda}$ of M which satisfy the following conditions:

1) For each λ, there is a submersion $f_\lambda\colon U_\lambda \to \mathbb{R}^q$ such that each connected component of $L_\alpha \cap U_\lambda$ is a connected component of a fiber of f_λ.
2) Let $\varphi_{\mu\lambda}$ be the transition function from U_λ to U_μ. Then, there exists a diffeomorphism $\gamma_{\mu\lambda}\colon p_\lambda(U_\lambda \cap U_\mu) \to p_\mu(U_\lambda \cap U_\mu)$ such that $\gamma_{\mu\lambda} \circ f_\lambda = f_\mu \circ \varphi_{\mu\lambda}$.

Such an atlas is called a *foliation atlas*. The integer q is called the (real) *codimension* of the foliation.

Remark 1.1.2.

1) We may assume that fibers of f_λ are homeomorphic, and U_λ is homeomorphic to $V_\lambda \times B_\lambda$ in a way such that f_λ is the projection to the second factor, where $B_\lambda = f_\lambda(U_\lambda)$ and V_λ is the fiber of f_λ.
2) If we assume that each f_λ is only continuous and that $\gamma_{\nu\mu}\gamma_{\mu\lambda} = \gamma_{\nu\lambda}$ for any $\lambda, \mu, \nu \in \Lambda$, then structures as above is called *Γ_q-structures*, where Γ_q denotes the pseudogroup of local diffeomorphisms of $\mathbb{R}^q$.