Bayesian Robustness
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SUFFICIENCY AND INFLUENCE[1]

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Consider two models M_1 and M_2 proposed as models for the same data. Assume that the conclusions from both models are posteriors $p_1(\theta|Y)$ and $p_2(\theta|Y)$ of some inferential target θ given the data Y. The unknowns θ may be parameters with identical interpretations in both models. Under mild conditions, the difference between the two conclusions is reducible to a one dimensional summary $h(\theta)$ for any two models. The result has implications for Bayesian diagnostics and sensitivity analysis. Applications of influence sufficiency to case and prior influence are illustrated, with emphasis on the influence of different priors and calculation of Bayes factors.

1. Introduction. I start with a brief discussion of influential and outlying observations, influential and unsupported assumptions and Bayesian robustness.

An observation is influential if the conclusion changes in an important manner when the observation's likelihood contribution changes. In contrast, outliers are observations whose responses differ from what is predicted by the model. Outliers need not be influential (consider either a model with t errors or regression through the origin) and influential observations need not be outliers; consider an observation in linear regression with leverage $h_i \approx 1$. As the number of observations increases, with all other aspects of the model held fixed, we can usually expect the influence of individual observations to become small.

Observations are not the only things in an analysis that can be influential. Assumptions such as linearity, normality, constant variance or smoothness contribute strongly to the likelihood. An assumption is influential if the conclusion changes when the assumption is relaxed. An assumption is outlying if the data support a relaxation of the assumption; usually we call this an unsupported assumption.

Classically, robustness has meant that within a range around a particular model, the derivatives of an inference, usually narrowly defined as a point estimate, with respect to various inputs are 'small'. Usually only the response variables are considered as inputs. From a Bayesian perspective, the classical definition of robustness can be encorporated into an analysis through choice of robust likelihoods (for example, Ramsay and Novick 1980). This is done through a priori beliefs and a posteriori data selection amongst models for the sampling density and not a blind requirement that models should be robust in the classical sense.

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