Part I
Two-dimensional chiral quantum fields

In Section 1, the notion of fields and the residue products are introduced algebraically and some elementary properties are summarized. Section 2 is devoted to the study of the mutual locality of fields. The notion of operator product expansion is explained and some examples are given. In Section 3, we will derive our identity (3) in the introduction.

1 Fields and their residue products

We describe the definitions and basic properties concerning two-dimensional chiral quantum fields in the language of formal Laurent series.

1.1 Preliminaries

Throughout the paper, we always work over a field $\mathbf{k}$ of characteristic zero. We denote by $V[[z, z^{-1}]]$ the set of all formal Laurent series in the variable z with coefficients in a vector space V possibly having infinitely many terms both of positive and of negative degree:

$$V[[z, z^{-1}]] = \left\{ \sum_{n=-\infty}^{\infty} v_n z^{-n-1} \,\middle|\, v_n \in V \right\}.$$

The subset consisting of all series with only finitely many terms of negative degree is denoted by

$$V((z)) = \left\{ \sum_{n=n_0}^{-\infty} v_n z^{-n-1} \,\middle|\, v_n \in V, n_0 \in \mathbb{Z} \right\}.$$

Similarly we write

$$V[[y, y^{-1}, z, z^{-1}]] = \left\{ \sum_{m,n=-\infty}^{\infty} v_{m,n} y^{-m-1} z^{-n-1} \,\middle|\, v_{m,n} \in V \right\},$$

$$V((y, z)) = \left\{ \sum_{m=m_0}^{-\infty} \sum_{n=n_0}^{-\infty} v_{m,n} y^{-m-1} z^{-n-1} \,\middle|\, v_{m,n} \in V, m_0, n_0 \in \mathbb{Z} \right\}.$$