Advanced Studies in Pure Mathematics 53, 2009
Advances in Discrete Dynamical Systems
pp. 1–9

On periodic points of 2-periodic dynamical systems

João Ferreira Alves

§1. Introduction and statement of the result

Motivated by a recent extension of Sharkovsky's theorem to periodic difference equations [1] (see also [4]), here we show that kneading theory can be useful in the study of the periodic structure of a 2-periodic nonautonomous dynamical system.

Since the notions of zeta function and kneading determinant will play a central role in this discussion, we start by recalling them.

Let X be a set and $f : X \to X$ a map. For each $n \in \mathbb{Z}^+$, denote by f^n the nth iterate of f, defined inductively by

$$f^1 = f \text{ and } f^{n+1} = f \circ f^n, \text{ for all } n \in \mathbb{Z}^+.$$

In what follows we assume that each iterate of f has finitely many fixed points. The Artin-Mazur zeta function of f is defined in [3] as the invertible formal power series

$$\zeta_f(z) = \exp \sum_{n \geq 1} \frac{\#\mathrm{Fix}(f^n)}{n} z^n,$$

where

$$\mathrm{Fix}(f^n) = \{x \in X : f^n(x) = x\}.$$

Naturally, this definition is a particular case of a more general definition, necessary for our purposes.

Let $f : Y \to X$ be a map, with $Y \subset X$. In this case the nth iterate of f is the map $f^n : Y_n \to X$ defined inductively by:

$$Y_1 = Y, f^1 = f$$

Received November 14, 2006.
Revised October 12, 2007.
Key words and phrases. Zeta functions, periodic points, kneading determinants.
Partially supported by FCT/POCTI/FEDER.