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An approach to the Cartan geometry II : CR manifolds

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Introduction

One of the prominent features in the post-Oka development of the several complex variables is the extensive use of the Cauchy-Riemann partial differential equations. We also note the development of the CR geometry induced on the boundary. This geometry is introduced by E. Cartan [3] in low dimensional cases. The general case is developed by N. Tanaka [9], S.-S. Chern-J. Moser [4], S. Webster [10], and D. Burns. Jr.-S. Shnider [1]. This geometry will be the vehicle to set the Cauchy-Riemann equation geometrically.

The CR geometry is a special case of the Cartan geometry, which is regarded as a deformation of the Klein's classical geometry. Namely, for each classical geometry given as a homogenous space G/H we have the Cartan geometries modeled after G/H. For example, Riemann geometry is modeled after the euclidean geometry, which is the quotient of the group of euclidean motions by the orthogonal group. On a space X we have a Cartan geometry modeled after G/H when we have (1) a principal H-bundle E formed by frames, i.e. ways to identify up to equivalence (infinitesimally up to certain order) its neighborhood with open sets in G/H. (2) A Cartan connection on E valued in the Lie algebra of G.

CR geometry may be regarded as the case of Cartan geometry when the homogenous space is the unit ball in complex euclidean space acted by the group of holomorphic automorphisms. We constructed CR geometry in [6] from the above view point. However, we did not construct the frame bundle directly. We first construct the bundle of the frames of the first (infinitesimal) order and then we prolong it to the frame bundle. In this paper, we construct CR geometry by defining frames directly. We also write down the normal CR Cartan connections and discuss its global aspect.

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