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Principal blocks with extra-special defect groups of order 27

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§1. Introduction

Let G be a finite group and p be a prime number. Let b be a p-block of G, P be a defect group of b and $k(b)$ (respectively, $l(b)$) be the number of irreducible ordinary characters (respectively, irreducible Brauer characters) in b. Suppose that

(1) *two blocks b and b' of finite groups G and G' respectively, have the common defect group P and their Brauer categories $Br_{b,p}(G)$ and $Br_{b',p}(G')$ are equivalent.*

(See [FH] for Brauer categories.) When we consider only principal p-blocks, their defect groups are Sylow p-subgroups and having the same Brauer category is equivalent to having the same p-local structure. See the definition in section 4 in [R] : Finite groups G and H have the same p-local structure if they have a common Sylow p-subgroup P such that whenever Q_1 and Q_2 are subgroups of P and $f : Q_1 \to Q_2$ is an isomorphism, then there is an element $g \in G$ such that $f(x) = x^g$ for all $x \in Q_1$ if and only if there is an element $h \in H$ such that $f(x) = x^h$ for all $x \in Q_1$.

Under condition (1) there is a question whether we have

$$k(b) = k(b') \text{ and } l(b) = l(b') \tag{2}$$

or not. We have a following conjecture.

Conjecture 1. *When b and b' are principal blocks satisfying condition* (1), *the equalities in* (2) *hold.*

When P is an abelian group, it is known that a block b of G and its Brauer correspondent $Br_P(b)$ in $N_G(P)$ have the same Brauer category

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