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# Analysis on Anticommuting Self-Adjoint Operators

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***Dedicated to Professor ShigeToshi Kuroda on the occasion of his 60th birthday***

## I. Introduction

Two *bounded* linear operators $A$ and $B$ in a Hilbert space $\mathcal{H}$ are said to anticommute if $AB + BA = 0$. However, if $A$ and $B$ are *unbounded*, then this definition of anticommutativity does not work, because $AB + BA$ may not make sense on any vector in $\mathcal{H}$.

A proper notion of anticommutativity of (*unbounded*) self-adjoint operators was given by Vasilescu [23]. Samoilenko [21] and Pedersen [20] gave several equivalent characterizations of the anticommutativity and discussed some aspects of anticommuting self-adjoint operators.

Following [20], we say that *two self-adjoint operators $A$ and $B$ in a Hilbert space anticommute if*

$$e^{itA}B \subset Be^{-itA}$$

*for all $t \in \mathbb{R}$.* We remark that this definition is symmetric in $A$ and $B$ [20] and gives an extension of the notion of anticommutativity of bounded operators mentioned above.

Families of anticommuting self-adjoint operators are not only interesting in its own right (in particular, from representation theoretical points of view), but also may be important in applications (e.g., analysis of operators of Dirac's type [3, 5-8, 13, 16] and supersymmetric quantum theory [1, 2, 4, 9, 15, 17, 18]).

In [10, 11] the present author has developed analysis on anticommuting self-adjoint operators; The paper [10] is concerned with algebraic properties of the partial isometries assoicated with anticommuting self-adjoint operators and analysis of the sum of two anticommuting

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