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Floer Homology for Oriented 3-Manifolds

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Dedicated to Professor Akio Hattori on his sixtieth birthday

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§1. Introduction

In [F], A. Floer introduced a new invariant for homology 3-spheres. In this paper we generalize his invariant to arbitrary closed and oriented 3-manifolds. In the case when the first homology group of the manifold is torsion free and nonzero, we also define invariants $I_k^s(M)$ for s < 3, which, in the case s = 0, is a generalization of Floer's one. The construction of this invariant is closely related also to the Donaldson's polynomial for closed 4-manifolds [D4]. The construction is based on the study of the moduli space of selfdual connections over $M \times \mathbf{R}$ and its compactification.

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